VICTORIA JUNIOR COLLEGE
Preliminary Examination

MATHEMATICS
(Higher 2)

9740/02

Paper 2

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF15)

September 2014

3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

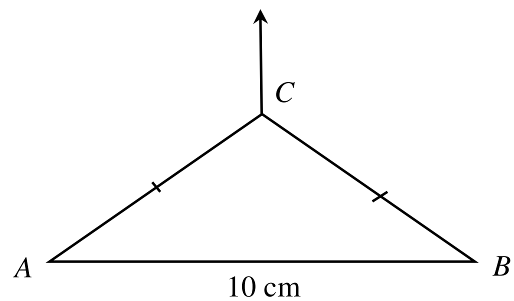
This document consists of 6 printed pages

Section A: Pure Mathematics [40 marks]

1 **(a)** It is given that the volume of an open right circular cone made of thin cardboard is a fixed value k cm^3, and the external surface area is a minimum. Use differentiation to find the height of the circular cone in terms of k. [5]

[The surface area S cm^2 of a circular cone with base radius r cm and height h cm is given by $S = \pi r\sqrt{r^2+h^2}$. Its volume V cm^3 is given by $V = \frac{1}{3}\pi r^2 h$.]

(b)



The diagram shows an isosceles triangle ABC with $AC = BC$. The points A and B are fixed and the vertex C is moving in a direction perpendicular to AB and away from AB with a constant speed of 2 cm per second. Calculate the rate of change of the angle ACB at the instant when triangle ABC is equilateral. [4]

2 **(a)** The line l passes through the point A with position vector $8\mathbf{i}+4\mathbf{j}+3\mathbf{k}$ and is parallel to the vector $\mathbf{i}+\mathbf{j}+2\mathbf{k}$. The non-coincident planes Π_1 and Π_2 are both perpendicular to l. The perpendicular distance from A to both planes is $3\sqrt{6}$. Find Cartesian equations for Π_1 and Π_2 . [4]

(b) The lines l_1 and l_2 have equations $\mathbf{r} = 2\mathbf{i}+\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})$ and $\mathbf{r} = 3\mathbf{i}+8\mathbf{j}+\mu(\mathbf{i}+2\mathbf{j}+a\mathbf{k})$ respectively, where λ , μ are parameters and a is a constant.

(i) Given that l_1 and l_2 are skew lines, what can be said about the value of a ? [3]

(ii) It is given that $a = -3$. The plane p contains l_1 and has no common point with l_2. Find an equation of p in scalar product form. [3]

3 **(i)** Find, in exact form $re^{i\theta}$, the three roots z_1, z_2 and z_3 of the equation

$$z^3 = -3\sqrt{3} + 3\text{i},$$

where $-\pi < \arg(z_1) < \arg(z_2) < \arg(z_3) < \pi$. [3]

(ii) Hence, find in exact form, the roots of the equation $w^3 = -3 - 3\sqrt{3}\text{i}$. [3]

(iii) Show the roots in part **(i)** on an Argand diagram. [2]

(iv) Given that b is a complex number and bz_2 is real and positive, find $\arg(b)$. [2]

4 **(i)** Given that $\dfrac{(n+1)}{n^2(n+2)^2} = a\left(\dfrac{1}{n^2} - \dfrac{1}{(n+2)^2}\right)$, find the value of the constant a. [2]

(ii) Show that $\displaystyle\sum_{n=2}^{N} \frac{(n+1)}{n^2(n+2)^2} = p + \frac{q}{(N+1)^2} + \frac{r}{(N+2)^2}$ where p, q and r are constants to be determined. [3]

(iii) Give a reason why the series $\displaystyle\sum_{n=2}^{N} \frac{(n+1)}{n^2(n+2)^2}$ converges as $N \to \infty$, and write down the value of the sum to infinity. [2]

(iv) Deduce the value of $\displaystyle\sum_{n=3}^{\infty}\left(\frac{1}{n!} + \frac{(n-1)}{n^2(n-2)^2}\right)$, giving your answer in an exact form.

[You may use the standard results given in the List of Formulae (MF15).] [4]

Section B: Statistics [60 marks]

5 There were 500 spectators seated at the grandstand of a stadium. A surveyor took the seat plan of the grandstand and threw a die on the seat plan 20 times and interviewed the spectators at the seats on which the die landed. Give a reason why this might not result in a random sample of spectators. [1]

In a second survey, the surveyor used a random number generator to obtain 20 distinct random numbers and interviewed the spectators at the seats corresponding to these numbers. Suggest how a more representative sample may be obtained by using this sampling method. [1]

A girl sold souvenirs to the spectators during the entire match. In the first 5 minutes of the match, she sold 2 souvenirs. State an assumption that is necessary for the number of souvenirs sold per hour to be modelled by a Poisson distribution. Explain if the assumption is valid in the context of this question. [2]

[Turn Over

6 When Mr Ong visits a fast food restaurant, he will choose either a Beef Combo Meal or a Chicken Combo Meal. During his first visit, the probability that he chooses a Beef Combo Meal is 0.8. Thereafter, the probability that he will choose a Beef Combo Meal is 0.6 if he chose one in his previous visit and 0.7 if he did not choose one in his previous visit.

(i) Show that the probability that Mr Ong chooses a Beef Combo Meal in each of the first three consecutive visits is 0.288. [1]

(ii) Find the probability that he chooses exactly one Beef Combo Meal in the first three consecutive visits. [2]

(iii) Mr Ong chose exactly two Chicken Combo Meals in his first three consecutive visits. Find the probability that he chose a Beef Combo Meal in his second visit. [2]

(iv) State, with a reason, if it is possible to use a binomial distribution B(5, p) to determine the probability that Mr Ong chooses four Beef Combo Meals in his first five consecutive visits. [1]

7 A glass of bourbon sling is produced by mixing two separate portions of bourbon whiskey with one portion of lemon juice. The portions, in ml, of bourbon whiskey and lemon juice are modelled as having normal distributions with means and standard deviations as shown below.

	Mean volume (in ml)	Standard deviation (in ml)
Bourbon whiskey	29.5	0.6
Lemon juice	30.2	0.4

Find the probability that the volume of a glass of bourbon sling lies between 87ml and 95ml. [3]

State an assumption needed for your calculation. [1]

Bourbon whiskey cost $50.00 per litre and lemon juice cost $3.00 per litre. A glass of bourbon sling is sold at $10.60. Find the probability that the profit for one glass of bourbon sling is at least $7.50. [4]

8 Non-steroidal anti-inflammatory drugs (NSAIDs), such as aspirin and ibuprofen, are used to treat inflammation, pain and fever. NSAIDs are associated with several adverse effects such as nausea, vomiting, dyspepsia – a form of gastrointestinal disorder, and in more serious instances, ulcers and bleeding in the stomach.

A doctor prescribes NSAIDs to 20 of his patients in a given week and the number of patients who developed dyspepsia is denoted by X. Studies have shown that 15% of patients develop dyspepsia. Assuming a binomial distribution for X, calculate,

(i) the probability that at least 15% of these 20 patients develop dyspepsia, [2]

(ii) the least integer n such that the probability of there being more than n patients who develop dyspepsia out of these 20 patients is less than 0.1. You must show sufficient working to justify your answer. [3]

In a clinical trial, 200 patients were selected for a research study on the effects of NSAIDs. The proportion of patients who develop dyspepsia remains as 15%.

(iii) Using a suitable approximation, calculate the probability at least 15% of these 200 patients develop dyspepsia. [3]
When the sample size gets very large, the probability that at least 15% of patients develop dyspepsia approaches k. State the value of k. [1]

9 A foreign exchange trader studied the daily movement of the rates of Australian Dollar (A$) and Singapore Dollar (S$) with respect to one US Dollar (US$) over 8 trading days. The closing rates of these two currencies for the 8 days are given below.

Day	1	2	3	4	5	6	7	8
S$ to 1 US$ (x)	1.2496	1.2499	1.2500	1.2501	1.2502	1.2499	1.2497	1.2503
A$ to 1US$ (y)	1.0572	1.0577	1.0579	1.0581	1.0583	1.0576	1.0572	1.0587

(i) Draw a scatter diagram for the data. [1]

(ii) Calculate the value of the product moment correlation coefficient between x and y, giving your answer correct to 4 decimal places. Suggest a reason why a linear model may not be the best model for the relationship between x and y. [2]

(iii) The trader used a different model given by the equation $y = ae^{bx} + 1$ to estimate the rate of S$ to 1 US$ when the rate of A$ to 1 US$ is 1.0582. Find the equation of a suitable regression line, and use it to find the required estimate, giving your answer to 4 decimal places. [4]

(iv) Explain how the trader can conclude that $y = ae^{bx} + 1$ is a better model compared to the linear model $y = c + dx$. [2]

(v) Comment on the use of the model in part **(iii)** in predicting the rate of S$ to 1 US$ when the rate of A$ to 1 US$ is less than 1. [1]

[Turn Over

10 A party of 12 people is to travel in 3 cars, with 4 people in each car. Each car is driven by its owner. If the arrangement of people within each car is not important, find the number of ways in which the party can be divided among the 3 cars if

(i) there are no restrictions, [2]

(ii) A and B, 2 of the 9 people who do not own the cars, refuse to travel in the same car. [3]

Upon reaching their destination, the party of 12 stand in a circle to play a game. Find the number of ways in which the 12 people can arrange themselves

(iii) if A and B refuse to stand next to each other, [3]

(iv) such that when A looks to his left, there are at least 3 people between he and B. [3]

11 **(a)** The mass of fruits in a randomly chosen fruit basket, Y g, has a normal distribution. The mass of an empty basket is 200g. A random sample of 9 filled fruit baskets is taken and the mass of each filled fruit basket, x g, is measured. The data are summarized by

$$\sum x = 10564, \qquad \sum x^2 = 12\,419\,788.$$

Write down an equation relating x and y, where y g denotes the mass of fruits in each fruit basket. Test, at the 5% level of significance, whether the mean mass of fruits in the fruit baskets is not 1 kg. [5]

State the meaning of "p-value" in the context of the question. [1]

(b) A random variable X has mean μ and variance 2. A sample of 50 random observations of X is taken.

(i) In a one-tail test of the null hypothesis $\mu = 10$, the sample mean is found to be 9.56 units and the alternative hypothesis is accepted. State the alternative hypothesis. Given that the significance level of the test is $100\alpha\%$, find an inequality satisfied by α. [3]

State, giving a reason, if any assumptions about the distribution of X are needed in order for the test to be valid. [1]

(ii) A student wants to investigate if the hypothesis $\mu = 10$ is incorrect. He carried out a z test at the 5% significance level and found sufficient evidence to conclude that $\mu \neq 10$. State, giving your reasons, whether it is necessarily true that the student would have found sufficient evidence to conclude that $\mu \neq 10$ if he had used a t test at the 5% significance level. [2]