VICTORIA JUNIOR COLLEGE
Preliminary Examination

MATHEMATICS
(Higher 2)

9740/01

Paper 1

Additional Materials: Answer Paper
Graph Paper
List of Formulae (MF15)

September 2014
3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CT group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of 6 printed pages

 VICTORIA JUNIOR COLLEGE **[Turn over**

1 The line $y = mx + \dfrac{3}{m}$ is a tangent to the parabola $y^2 = kx$ for all values of m where $m \neq 0$. Find the constant k and the coordinates of the point of contact between the two graphs, giving your answer in terms of m. [4]

2 The general solution to the differential equation

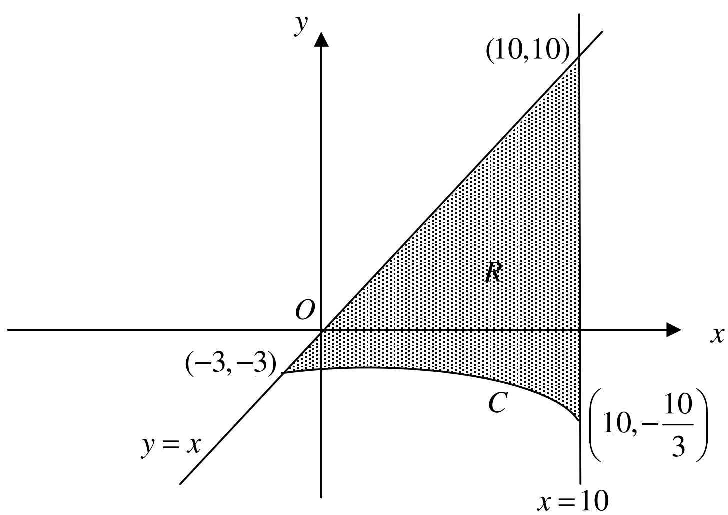
$$\frac{\mathrm{d}y}{\mathrm{d}x} = y - x + 2,$$

is given by $y = x + b + De^{x}$, where D is an arbitrary constant. Find the value of the constant b. [2]

Sketch, on a single diagram, three members of the family of solution curves, corresponding to $D > 0$, $D < 0$ and to $D = 0$. [3]

3 A curve C is defined by the parametric equations

$$x = 4t^2 - 24t - 3, \quad y = \frac{t+3}{t^2-1}, \qquad \text{for } -\frac{1}{2} \leq t \leq 0.$$



The shaded region R is bounded by C, the line $y = x$ and the line $x = 10$.

(i) Show that the area of R is $\dfrac{99}{2} + 32\ln 3$. [4]

Another curve D is defined by the parametric equations

$$x = \frac{t+3}{t^2-1}, \quad y = 4t^2 - 24t - 3, \qquad \text{for } -\frac{1}{2} \leq t \leq 0.$$

(ii) Deduce the area of the region bounded by C, D, the line $y = 10$ and the line $x = 10$. [1]

4 Prove by mathematical induction that

$$\sum_{r=1}^{n} a(2b)^r = \frac{2ab(1-(2b)^n)}{1-2b},$$

where a and b are constants such that $a \neq 0$ and $b \neq \frac{1}{2}$. [4]

Find the set of values of b such that the above series is convergent. State the sum to infinity of the series in terms of a and b. [2]

5 The arithmetic sequence $u_1, u_2, u_3, u_4, \ldots$ has first term 2.8 and common difference 2. The geometric sequence $v_1, v_2, v_3, v_4, \ldots$ has first term 3 and common ratio 1.2. Given that $w_n = u_n - v_n$ for $n = 1, 2, 3, \ldots$, find,

(i) w_n in terms of n, [2]

(ii) the set of values of n for which $w_n > 0$, [2]

(iii) the sum of all terms w_n for which $w_n > 0$, giving your answer in the form $p + q(1.2)^r$ where p, q, r are constants to be found. [3]

6 The complex number z satisfies the relations $|z-3-2\mathrm{i}| \leq \sqrt{13}$ and $|z| \geq |z-4\mathrm{i}|$. It is also given that $\mathrm{Re}(z) \geq 0$.

(i) On an Argand diagram, sketch the region R in which the point representing z can lie. [3]

(ii) Find, in an exact form, the set of values of $|z|$. [2]

(iii) Find the area of R, giving your answer to 3 decimal places. [3]

7 A curve C is defined by the equations

$$x = t^2, \quad y = 2t,$$

where t is a real parameter.

(i) Find the equation of the normal to C at the point P with parameter p. [3]

(ii) Given that this normal cuts C again at the point Q, and that $p = \sqrt{2}$, prove that the lines joining the origin to the points P and Q are perpendicular. [5]

8 Sketch the graph of $y=\dfrac{2x^2-11x+25}{x-1}$, stating the equations of any asymptotes and the coordinates of turning points and intersections with the axes, if appropriate. [You may leave the coordinates of the turning points correct to 3 significant figures.] [4]

On your diagram, label the point P with x-coordinate 2.

(i) The function f is defined by

$$\mathrm{f}: x \mapsto \frac{2x^2-11x+25}{x-1}, \quad x \geq 2.$$

(a) Giving a reason, determine if f^{-1} exists. [1]

(b) The function h is defined by

$$\mathrm{h}: x \mapsto \mathrm{f}(x), \quad x \geq \lambda.$$

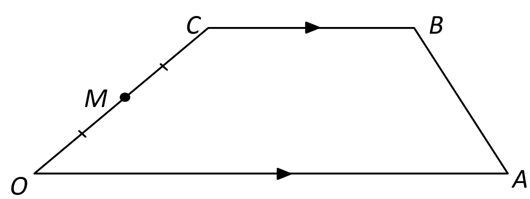
State the least integer value of λ for which h^{-1} exists. [1]

(ii) The function g is defined by

$$\mathrm{g}: x \mapsto x^3-9x^2+100, \quad x>4.$$

Find the range of gf. [2]

9 **(a)**



$OABC$ is a trapezium such that CB is parallel to OA and $CB : OA = k : 1$, where k is a constant and $0 < k < 1$.

It is given that $\overrightarrow{OA}=\mathbf{a}$, $\overrightarrow{OC}=\mathbf{c}$, and M is the midpoint of OC. Find $\overrightarrow{OB}$ in terms of k, $\mathbf{a}$ and $\mathbf{c}$ and show that the area of triangle AMB can be written as $\lambda|\mathbf{a}\times\mathbf{c}|$, where λ is a constant to be found in terms of k. [4]

(b) The points S and T have position vectors $2\mathbf{i}+3\mathbf{j}+2\mathbf{k}$ and $\mathbf{i}+4\mathbf{j}-3\mathbf{k}$ respectively. The plane p has equation $x+z=5$.

(i) Find the acute angle between p and the line passing through S and T. [3]

(ii) A second plane q is perpendicular to the vector $a\mathbf{i}+b\mathbf{j}+\mathbf{k}$, where a, b are constants and $a<-1$. Given that q is parallel to ST and the acute angle between p and q is 60°, find the value of a. [5]

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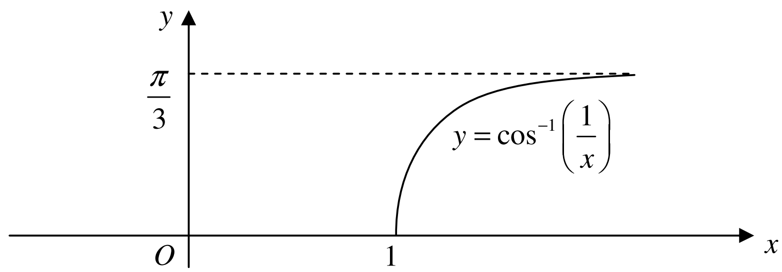


The diagram shows a sketch of the curve C with equation $y = \cos^{-1}\left(\frac{1}{x}\right)$ for $0 \le y \le \frac{\pi}{3}$. C is rotated about the y-axis to form the curved surface of an open container with a horizontal circular base of radius 1 unit. Given that $0 \le h \le \frac{\pi}{3}$, show that V, the volume of water inside the container when it is used to hold water up to a depth of h, is given by $V = \pi \tan h$. [3]

The container was initially full of water but a hole at the base causes water to leak at a rate inversely proportional to h, the depth of water at time t minutes. By first forming an equation relating $\frac{dV}{dt}$, $\frac{dh}{dt}$ and h, show that

$$h \sec^2 h \frac{dh}{dt} = -\frac{k}{\pi},$$

where k is a positive constant. [3]

Given that it takes half an hour for the water to completely drain out, find the value of k. [4]

11 Let $f(x) = \frac{2\cos 2x}{1+\sin 2x}$.

(i) Sketch the graph of $y = f(x)$ for $-0.5 \le x \le 0.5$. [2]

(ii) Given that x is small and x is in radians, find, without differentiation, the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [3]

(a) State the equation of the tangent to the curve $y = f(x)$ at the point (0, 2). [1]

(b) If the term in x^n in the expansion of $f(x)$ has coefficient k, find the value of $\frac{d^n y}{dx^n}$ at $x = 0$, giving your answer in terms of n and k. [2]

Denote the expansion of $f(x)$ in part **(ii)** by $g(x)$.

(iii) By substituting $x = \frac{\pi}{12}$ in $g(x)$, find an approximation for $\sqrt{3}$, giving your answer in the form $a + b\pi + c\pi^2$, where a, b, c are constants to be found. Suggest how the error in the approximation can be reduced. [3]

(iv) Find, for $-0.5 < x < 0.5$, the set of values of x for which the value of $g(x)$ is within ± 0.5 of the value of $f(x)$. [3]

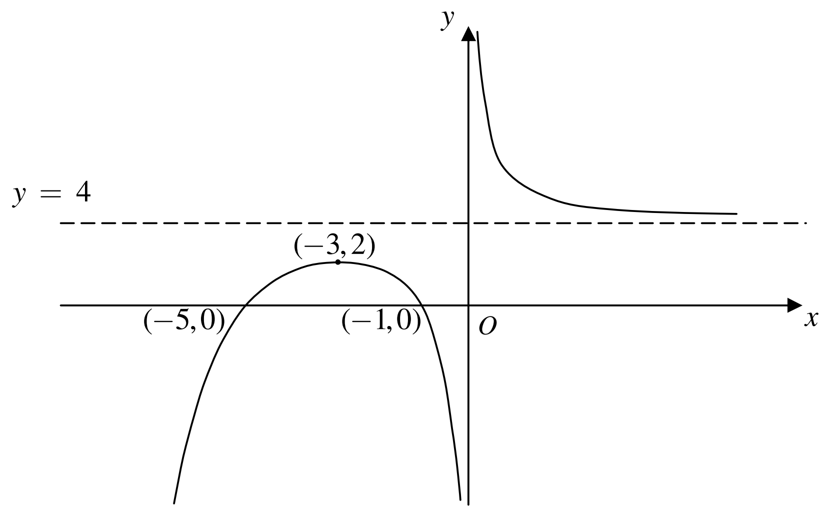
12 (**a**) A curve C has equation $x^2+6x+5=\dfrac{(y-5)^2}{k}$, where k is a constant.

(i) Given that $k=-4$, sketch C, making clear the main relevant features. [3]

(ii) Describe fully a sequence of **three** transformations which would transform the curve sketched in part **(i)** onto the curve $x^2+y^2=16$. [3]

(iii) State the set of all possible values of k for which C would have 2 asymptotes. [1]

(b)



The diagram shows a sketch of the curve $y=\mathrm{f}(x)$. The y-axis and the line $y=4$ are asymptotes to the curve. The curve intersects the x-axis at the points $(-5,\ 0)$, $(-1,\ 0)$ and has a maximum point at $(-3,\ 2)$.

On separate diagrams, draw sketches of the graphs of

(i) $y=\mathrm{f}'(x)$, [3]

(ii) $y=\dfrac{1}{\mathrm{f}(x)}$, [3]

stating the equations of any asymptotes, coordinates of turning points and intersections with the axes, if appropriate.