

YISHUN JUNIOR COLLEGE
2014 JC2 PRELIMINARY EXAMINATION

MATHEMATICS

9740/02

Higher 2

Paper 2

20 AUGUST 2014

WEDNESDAY 1400h – 1700h

Additional materials:

Answer paper

List of Formulae (MF15)



TIME 3 hours

READ THESE INSTRUCTIONS FIRST

Write your name and CTG in the spaces provided on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, write down the question number of the questions attempted, model of calculator used on the spaces provided on the cover page. Tie your cover page on top of the answer scripts before submission.

The number of marks is given in brackets [] at the end of each question or part question.

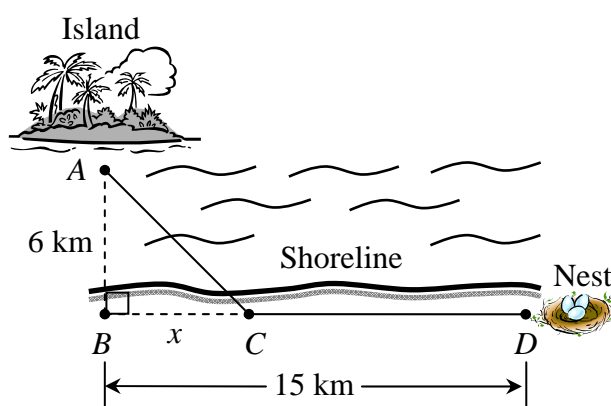
Section A: Pure Mathematics [40 marks]

- 1 On an Argand diagram, sketch the locus of the points representing the complex number z such that $|z - 2 + i| = \sqrt{5}$. [2]

Hence find exactly the range of

- (i) $|1 - z|$, [3]
 (ii) $\arg(iz - 1 + 3i)$. [3]

- 2 A bird is released from point A at an island that is 6 km from the nearest point B on a shoreline. It flies to a point C on the shoreline, and then flies along the shoreline to its nest D . Points B and D are 15 km apart and let the distance BC be x km.



If W and L are the energy (in joules) per kilometre when flying over water and land respectively, show that the total energy expenditure E is given by

$$E = W\sqrt{36 + x^2} - Lx + 15L. \quad [1]$$

- (i) If the bird takes 1.4 times as much energy to fly over water as land, determine the value of x that minimises the total energy expenditure. [4]

Assume that a bird of another species instinctively chooses a path that will minimise its energy expenditure.

- (ii) If it is observed that the bird reaches the shore at C which is 4 km from B , find the value of $\frac{W}{L}$. [2]

- (iii) State the implication of the bird's flight path if the ratio $\frac{W}{L}$ is large. [1]

3 (a) Find $\int \frac{1}{4+9x^2} dx$. [2]

(b) Find the exact value of $\int_0^2 \frac{x}{4+9x^2} dx$. [3]

(c) Sketch the graph of $y = e^{\sqrt{x}}$ where $x \geq 0$. [1]

The region bounded by the curve $y = e^{\sqrt{x}}$, the line $x = 4$ and the x - and y - axes is rotated completely about the x -axis. Use the substitution $t = \sqrt{x}$ to show that the volume of the solid obtained is given by $2\pi \int_0^2 te^{2t} dt$, and evaluate this integral exactly. [6]

4 Relative to the origin O , the points A , B and C have position vectors given by $\mathbf{i} - \mathbf{k}$, $3\mathbf{i} + \mathbf{j} - \mathbf{k}$ and $\mathbf{j} + 2\mathbf{k}$ respectively.

(i) Show that a vector equation of the plane π_1 containing A , B and C is given by $\mathbf{r} \cdot (\mathbf{i} - 2\mathbf{j} + \mathbf{k}) = 0$. [3]

The line l_1 passes through the points A and B . The point R on l_1 is such that CR is perpendicular to l_1 . The line l_2 passes through C and R and the point Q on l_2 is such that $2\overrightarrow{CR} = \overrightarrow{CQ}$.

(ii) Find the position vector of R . [4]

(iii) Find the exact area of the triangle BCQ . [3]

(iv) Find an equation in the form $\mathbf{r} \cdot \mathbf{n} = p$, for the plane π_2 which contains the line l_2 and is perpendicular to π_1 . [2]

Section B: Statistics [60 marks]

5 A college has 3000 students in the Arts, Science and Commerce stream. To gather feedback on the revised timetable, a teacher wishes to obtain a random sample of 120 students.

(i) Explain what is meant in this context by the term ‘a random sample’ and why this may not provide a representative sample. [2]

(ii) Name a more appropriate sampling method to provide a representative sample and explain how it can be carried out. [2]

- 6 A mother monitored the growth of her baby and recorded the height h cm and weight y kg at various stages in the baby development. The results were as follows.

h	50	58	63	68	82	88	96
y	3.93	4.38	5.81	6.68	10.13	13.10	17.45

The mother thought that a model of the form $y = p + qh$, where p and q are constants, might be suitable to describe the relationship between y and h .

- (i) Draw a scatter diagram to illustrate the data. [1]
- (ii) Calculate the value of the product moment correlation coefficient. Explain why its value does not necessarily mean that the best model for the relationship between y and h is $y = p + qh$. [2]
- (iii) Explain which of the following would be the best model to represent the above data.
 (A) $y = a + be^h$
 (B) $y = c + dh^3$ [2]
- (iv) It is required to estimate the weight of the baby when his height is 75 cm. Find the equation of a suitable regression line, and use it to find the required estimate. Comment on the reliability of the estimation. [3]
- 7 (a) The random variable X has a binomial distribution with mean 4 and variance $\frac{4}{3}$.
 Show that the largest value which X can take is 6. [2]
- A sample of N (> 50) independent observations of X is taken. Find the least value of N such that there is a probability of more than 0.9 for the sample mean to be at least 3.8. [3]
- (b) In a study of a certain species of tigers, a sample of 120 cubs is observed. The number of cubs which will reach its matured age is denoted by Y . State, in context, two assumptions needed for Y to be well modelled by a binomial distribution. [2]
- Assume now that Y has a binomial distribution and that the probability for a cub to reach its matured age is $\frac{3}{4}$. Using a suitable approximation, find the probability that at least 90 of these cubs will reach their matured age. [3]

- 8** An owner of a small factory uses a machine to dispense milk into bottles. The volume of milk dispensed is assumed to be a normal variable with a standard deviation of 8.5 ml.

- (i) The mean volume, k ml, of milk dispensed into a bottle can be altered on the machine. Regulations stipulate that more than 99% of the bottles must contain at least 750 ml of milk. Find the least value of k . [3]
- (ii) A random sample of 100 bottles of milk is chosen and a test is carried out at the 5% significance level, to determine whether the mean volume of milk dispensed is less than 750 ml. Find the range of values of the mean volume, \bar{x} ml, of the 100 bottles of milk for which the null hypothesis would be rejected. [4]
- (iii) The owner is interested in buying a new machine which the manufacturer claims that it can dispense a mean volume of 750 ml. A random sample of 18 bottles is taken and the volume of milk in each bottle, y ml, is measured. The results are summarised by

$$\sum y = 13581, \quad \sum y^2 = 10\,247\,700.$$

Assuming that the volumes dispensed by this machine are normally distributed, test at the 1% significance level, whether the manufacturer's claim is justified. [5]

- 9** (i) State two conditions needed for the number of typing errors on a randomly chosen page in a novel to be well modelled by a Poisson distribution. [2]

Assume that the number of typing errors on a page in the novel has a Poisson distribution with mean 0.4.

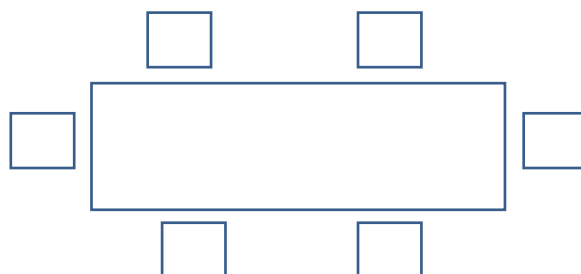
- (ii) Find the probability that on 3 randomly chosen pages, there is exactly one page containing at least one error. [2]
- (iii) Find the largest value of n for which there is a probability of at least 0.1 that there is exactly one error on the first n pages of the novel. [2]

Independently, the number of typing errors on a page, Y , on a Mathematics textbook also has a Poisson distribution.

- (iv) Given that $P(Y = 2) = 2 P(Y = 3)$, find the mean of Y . [2]
- (v) Using a suitable approximation, find the probability that there are at least four times as many typing errors on the first 100 pages of the Mathematics textbook as typing errors on the first 100 pages of the novel. [4]

- 10 (a)** For events A and B , it is given that $P(A) = 0.7$, $P(B / A') = 0.4$ and $P(A \cap B) = 0.28$.
- (i)** Determine, with a reason whether the events A and B are independent. [2]
 - (ii)** Find $P[(A \cap B) | (A \cup B)]$. [3]
- (b)** A committee of six people is to be selected from 4 single women and 4 single men and a married couple.
- (i)** Find the probability that a randomly chosen committee consists of either the wife *or* the husband *or* both. [2]
 - (ii)** Find the probability that there are more men than women in the committee, given that the committee includes at least two men. [3]

A committee of two men, two women and the married couple is formed. During a meeting, they are seated at a rectangular table with the chairs as arranged in the diagram below.



Find the probability that

- (iii)** the men and women must alternate, [2]
- (iv)** the couple sit directly opposite each other. [2]

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