TPJC 2014 JC2 Preliminary Examination
H2 Mathematics Paper 1 Solution

1 **(4m)**	At the points of intersection, $\sqrt{a}\,x + bx^2 = \ln(cx)$ When x =1, $\sqrt{a} + b - \ln c = 0$ ------------ (1) When x =2, $2\sqrt{a} + 4b - \ln c = \ln 2$ ------------ (2) When x =3, $3\sqrt{a} + 9b - \ln c = \ln 3$ ------------ (3) Using GC, $\sqrt{a} = 1.124670 \quad \Rightarrow \quad a = (1.124670)^2 = 1.265$ (to 3 d.p.) $b = -0.144$ (to 3 d.p.) $\ln c = 0.98083 \quad \Rightarrow \quad c = \mathrm{e}^{0.98083} = 2.667$ (to 3 d.p.)

2 **(5m)**	Let P_n be the statement "$u_n = \frac{2}{3}\left(4^n - 1\right), n \in \mathbb{Z}^+$." When $n = 1$, $u_1 = \frac{2}{3}\left(4^1 - 1\right) = \frac{2}{3}(3) = 2$ (given). $\therefore \mathrm{P}_1$ is true. Assume P_k is true for some $k \in \mathbb{Z}^+$. When $n = k$, $u_k = \frac{2}{3}\left(4^k - 1\right)$ To prove P_{k+1} is true, ie. $u_{k+1} = \frac{2}{3}\left(4^{k+1} - 1\right)$. When $n = k + 1$, $u_{k+1} = 4u_k + 2$ $= 4\left[\frac{2}{3}\left(4^k - 1\right)\right] + 2$ $= \frac{2}{3}(4)\left(4^k\right) - \frac{8}{3} + 2$ $= \frac{2}{3}\left(4^{k+1}\right) - \frac{2}{3}$ $= \frac{2}{3}\left(4^{k+1} - 1\right)$ $\therefore \mathrm{P}_k$ is true $\Rightarrow \mathrm{P}_{k+1}$ is true. Since P_1 is true, P_k is true $\Rightarrow \mathrm{P}_{k+1}$ is true, by Mathematical Induction, P_n is true $\forall\, n \in \mathbb{Z}^+$.

3(i) **(6m)**	$\lvert a \rvert = 1$ $\sqrt{(3p)^2 + (-2p)^2 + (6p)^2} = 1$ $49p^2 = 1$ $p = \frac{1}{7} \quad , \quad p > 0$
(ii)	Length of projection of *OB* along *OA*
(iii)	$(\mathbf{b}-\mathbf{a}).(\mathbf{b}-\mathbf{a}) = \lvert\mathbf{b}\rvert^2 + \lvert\mathbf{a}\rvert^2 - 2(\mathbf{a.b})$ $\lvert\mathbf{b}\rvert^2 + \lvert\mathbf{a}\rvert^2 - (\mathbf{b}-\mathbf{a}).(\mathbf{b}-\mathbf{a}) = 2(\mathbf{a.b})$ $= 2p\begin{pmatrix} 3 \\ -2 \\ 6 \end{pmatrix}.\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ $= \frac{2(6+2+12)}{7} = \frac{40}{7}$

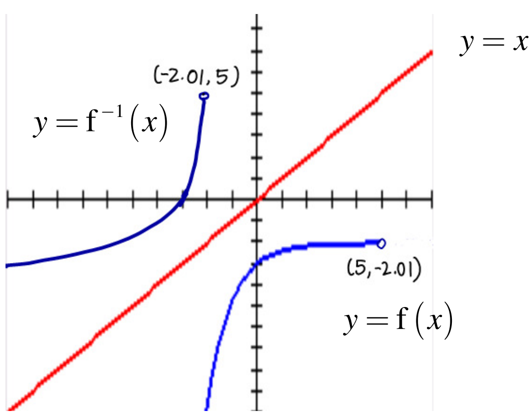
4(a) **(7m)**	$u_{n+1} - u_n = 4 - \frac{5}{u_n + 2} - u_n$ $= 4 - \frac{{u_n}^2 + 2u_n + 5}{u_n + 2}$ $= 4 - \frac{(u_n + 1)^2 + 4}{u_n + 2}$ Strictly increasing $\Rightarrow u_{n+1} - u_n > 0$ $4 - \frac{(u_n + 1)^2 + 4}{u_n + 2} > 0$ $\frac{(u_n + 1)^2 + 4}{u_n + 2} < 4$ $(u_n + 1)^2 + 4 < 4(u_n + 2)$ since $u_n > 0$ ${u_n}^2 - 2u_n - 3 < 0$ $(u_n - 3)(u_n + 1) < 0$ $-1 < u_n < 3$ $\therefore$ the range of values is $0 < u_n < 3$.

(b)	$\mathrm{f}(r)-\mathrm{f}(r-1)=r(r+1)!-(r-1)r!$ $=r!\left(r(r+1)-(r-1)\right)$ $=r!\left(r^2+1\right)$ $\sum_{r=1}^{N} r!\left(r^2+1\right)=\sum_{r=1}^{N}\left(\mathrm{f}(r)-\mathrm{f}(r-1)\right)$ $=\mathrm{f}(1)-\mathrm{f}(0)$ $+\mathrm{f}(2)-\mathrm{f}(1)$ $+\ldots$ $+\mathrm{f}(N-1)-\mathrm{f}(N-2)$ $+\mathrm{f}(N)-\mathrm{f}(N-1)$ $=\mathrm{f}(N)-\mathrm{f}(0)$ $=(N)(N+1)!$

5(i) **(8m)**	$u=\sqrt{x-1}\Rightarrow\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{2\sqrt{x-1}}=\frac{1}{2u}$ $\int x\sqrt{x-1}\,\mathrm{d}x=\int\left(u^2+1\right)u(2u)\,\mathrm{d}u$ $=2\int\left(u^4+u^2\right)\mathrm{d}u$ $=2\left(\frac{u^5}{5}+\frac{u^3}{3}\right)+C$ $=\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}+C$
(ii)	$\int_1^2 x^2\,(x-1)^{\frac{1}{2}}\,\mathrm{d}x$ $=\left[x\left[\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}\right]\right]_1^2-\int_1^2\left[\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}\right]\mathrm{d}x$ $=\left[x\left[\frac{2}{5}(x-1)^{\frac{5}{2}}+\frac{2}{3}(x-1)^{\frac{3}{2}}\right]\right]_1^2-\left[\frac{4}{35}(x-1)^{\frac{7}{2}}+\frac{4}{15}(x-1)^{\frac{5}{2}}\right]_1^2$ $=2\left[\frac{2}{5}+\frac{2}{3}\right]-\left[\frac{4}{35}+\frac{4}{15}\right]$ $=\frac{184}{105}$

6(a) (8m)	$\dfrac{1-\mathrm{i}\tan\theta}{1+\mathrm{i}\tan\theta}=\dfrac{1-\mathrm{i}\dfrac{\sin\theta}{\cos\theta}}{1+\mathrm{i}\dfrac{\sin\theta}{\cos\theta}}$ $=\dfrac{\cos\theta-\mathrm{i}\sin\theta}{\cos\theta+\mathrm{i}\sin\theta}$ $=\dfrac{\mathrm{e}^{-\mathrm{i}\theta}}{\mathrm{e}^{\mathrm{i}\theta}}$ $=\mathrm{e}^{-\mathrm{i}(2\theta)}$ (shown) Alternatively, $\left\|\dfrac{1-\mathrm{i}\tan\theta}{1+\mathrm{i}\tan\theta}\right\|=\dfrac{\sqrt{1^2+(-\tan\theta)^2}}{\sqrt{1^2+(\tan\theta)^2}}=1$ $\arg\left(\dfrac{1-\mathrm{i}\tan\theta}{1+\mathrm{i}\tan\theta}\right)=\arg(1-\mathrm{i}\tan\theta)-\arg(1+\mathrm{i}\tan\theta)$ $=-\tan^{-1}(\tan\theta)-\tan^{-1}(\tan\theta)=-2\theta$ $\dfrac{1-\mathrm{i}\tan\theta}{1+\mathrm{i}\tan\theta}=\mathrm{e}^{-\mathrm{i}(2\theta)}$
(b)(i)	$z^3=(1+2a\mathrm{i})^3$ $=1+3(2a\mathrm{i})+3(2a\mathrm{i})^2+(2a\mathrm{i})^3$ $=1-12a^2+(6a-8a^3)\mathrm{i}$ Since z^3 is real, $6a-8a^3=0$ $2a(3-4a^2)=0$ Since $a\neq 0$, $3-4a^2=0$ $a^2=\dfrac{3}{4}$ $a=\pm\dfrac{\sqrt{3}}{2}$ $\therefore z=1+2\left(\dfrac{\sqrt{3}}{2}\right)\mathrm{i}$ or $z=1+2\left(-\dfrac{\sqrt{3}}{2}\right)\mathrm{i}$ $z=1+\sqrt{3}\,\mathrm{i}$ or $z=1-\sqrt{3}\,\mathrm{i}$

(b)(ii)	$\arg\left(\frac{p^2}{w^4}\right)=\frac{\pi}{12}$ $\arg\left(\frac{\left(zw^*\right)^2}{w^4}\right)=\frac{\pi}{12}$ $2\left(\arg z+\arg w^*\right)-4\arg w=\frac{\pi}{12}$ $2\left(\arg z-\arg w\right)-4\arg w=\frac{\pi}{12}$ $2\arg z-6\arg w=\frac{\pi}{12}$ $2\arg\left(1-\sqrt{3}\,\mathrm{i}\right)-6\theta=\frac{\pi}{12}$ $2\left(-\frac{\pi}{3}\right)-6\theta=\frac{\pi}{12}$ $6\theta=-\frac{2\pi}{3}-\frac{\pi}{12}=-\frac{3\pi}{4}$ $\theta=-\frac{3\pi}{24}=-\frac{\pi}{8}$

7(i) **(9m)**	 The graph of $y=\mathrm{f}(x)$ does not intersect the graphs of $y=\mathrm{f}^{-1}(x)$ and $y=x$ at any point. Hence there are no real values of x which satisfy $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$.
(ii)	Stretch, factor h, parallel to the x-axis Translate $\begin{pmatrix}0\\k\end{pmatrix}$ OR Translation of k units in the positive y direction

	Alternatively, Stretch, factor $\dfrac{1}{h}$, parallel to the y-axis Translate $\begin{pmatrix}0\\k\end{pmatrix}$ OR Translation of k units in the positive y direction Alternatively, Translate $\begin{pmatrix}-k\\0\end{pmatrix}$ OR Translation of k units in the negative x direction Stretch, factor h, parallel to the x-axis
(iii)	$\mathrm{gf}(x) = \mathrm{g}\left(-\mathrm{e}^{-x}-2\right)$ $= \dfrac{-\mathrm{e}^{-x}-2}{h} + k$ $\mathrm{gf}: x \to \dfrac{-\mathrm{e}^{-x}-2}{h} + k, \quad x \in \mathbb{R}, x < 5$
(iv)	For $\mathrm{gf}(x) = (\mathrm{gf})^{-1}(x)$, graph of $y = \mathrm{gf}(x)$ has to cut the graph of $y = x$ at at least one point. Suppose $y = x$ is a tangent to the graph of $y = \mathrm{gf}(x)$. Let $y = \dfrac{2}{3}\left(-\mathrm{e}^{-x}-2\right)$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2\mathrm{e}^{-x}}{3}$ $\dfrac{2\mathrm{e}^{-x}}{3} = 1$ $x = -\ln\dfrac{3}{2}$ $y = -\dfrac{7}{3}$ Translate point $\left(-\ln\dfrac{3}{2}, -\dfrac{7}{3}\right)$ to $\left(-\ln\dfrac{3}{2}, -\ln\dfrac{3}{2}\right) \Rightarrow k = \dfrac{7}{3} - \ln\dfrac{3}{2}$ $k \geq \dfrac{7}{3} - \ln\dfrac{3}{2}$

Least integer $k = 2$

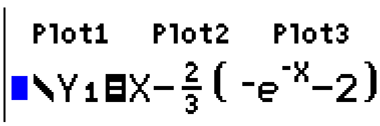
<u>Alternatively</u>

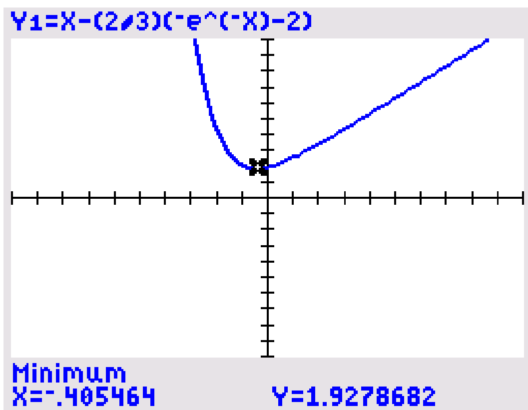
$\text{gf}(x) = x$

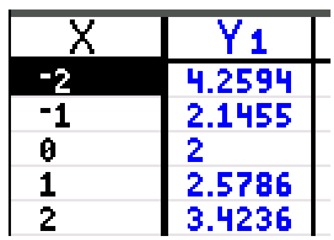
$\frac{2}{3}\left(-e^{-x} - 2\right) + k = x$

$k = x - \frac{2}{3}\left(-e^{-x} - 2\right)$

Using G.C.,







X	Y1
-2	4.2594
-1	2.1455
0	2
1	2.5786
2	3.4236

$k \geq 1.928$

Least integer $k = 2$

8 (i) (12m)

Direction vector of l is $\begin{pmatrix} -2 \\ 6 \\ 4 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$

Normal vector of p is $\begin{pmatrix} 1 \\ -3 \\ -2 \end{pmatrix}$

Since l is parallel to normal vector of p, l is perpendicular to p.

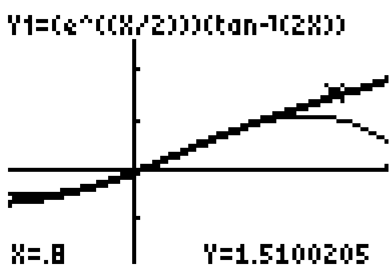
(ii)	l: $\boldsymbol{r}=\begin{pmatrix}-4\\4\\-1\end{pmatrix}+\lambda\begin{pmatrix}-2\\6\\4\end{pmatrix}$, $\lambda\in\mathbb{R}$ p: $\boldsymbol{r}.\begin{pmatrix}1\\-3\\-2\end{pmatrix}=0$ Substitute $\boldsymbol{r}=\begin{pmatrix}-4\\4\\-1\end{pmatrix}+\lambda\begin{pmatrix}-2\\6\\4\end{pmatrix}$ into $\boldsymbol{r}.\begin{pmatrix}1\\-3\\-2\end{pmatrix}=0$ $\begin{pmatrix}-4-2\lambda\\4+6\lambda\\-1+4\lambda\end{pmatrix}.\begin{pmatrix}1\\-3\\-2\end{pmatrix}=0$ $(-4-2\lambda)-3(4+6\lambda)-2(-1+4\lambda)=0$ $\lambda=-\dfrac{1}{2}$ Position vector of point of intersection = $\begin{pmatrix}-4-2(-\frac{1}{2})\\4+6(-\frac{1}{2})\\-1+4(-\frac{1}{2})\end{pmatrix}=\begin{pmatrix}-3\\1\\-3\end{pmatrix}$ Coordinates of point of intersection is $(-3,1,-3)$
(iii)	Equate $\begin{pmatrix}-10\\22\\11\end{pmatrix}=\begin{pmatrix}-4\\4\\-1\end{pmatrix}+\lambda\begin{pmatrix}-2\\6\\4\end{pmatrix}$ $-4-2\lambda=-10$ ---- (1) $4+6\lambda=22$ ---- (2) $-1+4\lambda=11$ ---- (1) $\lambda=3$ satisfies all three equations. *A* lies on *l*. Since $(-3,1,-3)$ is the mid-point of *A* and *B*, By Ratio Theorem, $\dfrac{\boldsymbol{OA}+\boldsymbol{OB}}{2}=\begin{pmatrix}-3\\1\\-3\end{pmatrix}$

	$\boldsymbol{OB} = 2\begin{pmatrix}-3\\1\\-3\end{pmatrix} - \begin{pmatrix}-10\\22\\11\end{pmatrix} = \begin{pmatrix}4\\-20\\-17\end{pmatrix}$ Coordinates of B is $(4,-20,-17)$
(iv)	Area of $OAB = \frac{1}{2}\left\|\boldsymbol{OA}\times\boldsymbol{OB}\right\|$ $= \frac{1}{2}\left\|\begin{pmatrix}-10\\22\\11\end{pmatrix}\times\begin{pmatrix}4\\-20\\-17\end{pmatrix}\right\|$ $= \frac{1}{2}\left\|\begin{pmatrix}-154\\-126\\112\end{pmatrix}\right\|$ $= \frac{1}{2}\sqrt{52136} = 114.166545$ $= 114 \text{ units}^2$

9(i) **(12m)**	$y = \frac{n-2x}{2} = \frac{n}{2} - x$ Volume = $(n-2x)yx$ $= (n-2x)\left(\frac{n}{2}-x\right)x$ $= \frac{n^2}{2}x - nx^2 - nx^2 + 2x^3$ $= \frac{1}{2}x(n^2 - 4nx + 4x^2)$ (Shown)
(ii)	$\frac{dV}{dx} = \frac{1}{2}n^2 - 4nx + 6x^2$ $\frac{dV}{dx} = 0 \Rightarrow \frac{1}{2}n^2 - 4nx + 6x^2 = 0$ $\Rightarrow 12x^2 - 8nx + n^2 = 0$ $(6x-n)(2x-n) = 0$ $x = \frac{n}{2}$ or $\frac{n}{6}$ If $x = \frac{n}{2}$, $2x = n$ which is not possible since $2x < n$ for the box to be possible. If $x = \frac{n}{6}$, $2x = \frac{n}{3} < n$ $\therefore x = \frac{n}{6}$ is the only answer

(iii)	$\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=12x-4n$ When $x=\frac{n}{6}$, $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=12\left(\frac{n}{6}\right)-4n=-2n<0$ $\therefore$ V is max when $x=\frac{n}{6}$ Max $V= =\frac{1}{2}\left(\frac{n}{6}\right)\left(n^2-4n\left(\frac{n}{6}\right)+4\left(\frac{n}{6}\right)^2\right)=\frac{1}{27}n^3$

10(i) **(13m)**	$y=\tan^{-1}(2x)$ Differentiate w.r.t x $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2}{1+(2x)^2}=\frac{2}{1+4x^2}$ Differentiate w.r.t x $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{-2(8x)}{(1+4x^2)^2}=(-4x)\left(\frac{2}{1+4x^2}\right)^2$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-4x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+4x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2=0$ (shown) Differentiate w.r.t x $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+4\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+8x\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)=0$ When $x=0$, $y=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=2$, $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=0$, $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-16$ $y=2x-16\left(\frac{x^3}{6}\right)+\cdots$ $y=2x-\frac{8}{3}x^3+\cdots$
(ii)	$e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right]$ $\approx\left[1+\frac{x}{2}+\frac{1}{2}\left(\frac{x}{2}\right)^2\right]\left(2x-\frac{8}{3}x^3\right)$ $\approx 2x+x^2+\frac{1}{4}x^3-\frac{8}{3}x^3$

	$= 2x + x^2 - \frac{29}{12}x^3$
(iii)	$\int_0^k e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right] \mathrm{d}x$ $\approx \int_0^k 2x + x^2 - \frac{29}{12}x^3 \ \mathrm{d}x$ $= \left[x^2 + \frac{x^3}{3} - \frac{29}{12}\left(\frac{x^4}{4}\right)\right]_0^k$ $= k^2 + \frac{k^3}{3} - \frac{29}{48}k^4$ When $k = 0.8$ $\int_0^{0.8} e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right] \mathrm{d}x \approx (0.8)^2 + \frac{(0.8)^3}{3} - \frac{29}{48}(0.8)^4 = 0.5632$
(iv)	$\int_0^{0.8} e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right] \mathrm{d}x = 0.638$ (to 3 s.f)  At $k = 0.8$, the two graphs $y = e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right]$ and $y = 2x + x^2 - \frac{29}{12}x^3$ have diverged from each other significantly. Therefore, the approximation in part **(iii)** is not very good.

11(i) **(16m)**	$x = \theta - \sin\theta$, $\quad y = 1 - \cos\theta$ $\frac{\mathrm{d}x}{\mathrm{d}\theta} = 1 - \cos\theta$, $\quad \frac{\mathrm{d}y}{\mathrm{d}\theta} = \sin\theta$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}\theta} \times \frac{\mathrm{d}\theta}{\mathrm{d}x}$ $= \frac{\sin\theta}{1 - \cos\theta}$

	$=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{1-\left(1-2\sin^2\frac{\theta}{2}\right)}$ $=\dfrac{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}{2\sin^2\frac{\theta}{2}}$ $=\dfrac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\cot\dfrac{1}{2}\theta$ (shown) When $\theta=\pi$, gradient at $C=0$. As $\theta\to 0$ or $\theta\to 2\pi$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}\to\infty$. Tangents to C are vertical lines.
(ii)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=-1$ $\cot\dfrac{1}{2}\theta=-1$ $\dfrac{1}{\tan\frac{1}{2}\theta}=-1$ $\tan\dfrac{1}{2}\theta=-1$ From the curve C, for $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-1$, $\pi<\theta<2\pi$ $\dfrac{1}{2}\theta=\dfrac{3\pi}{4}$ $\theta=\dfrac{3\pi}{2}$ $x=\dfrac{3\pi}{2}-\sin\dfrac{3\pi}{2}=1+\dfrac{3\pi}{2}$

	$y = 1 - \cos\frac{3\pi}{2} = 1$ Coordinates of M is $\left(1+\frac{3\pi}{2}, 1\right)$.
(iii)	Area of region between C and L $= \int_{1+\frac{3\pi}{2}}^{4+\frac{3\pi}{2}} 2 - \sqrt{x - \frac{3\pi}{2}}\, \mathrm{d}x - \int_{1+\frac{3\pi}{2}}^{2\pi} y\, \mathrm{d}x$ $= \int_{1+\frac{3\pi}{2}}^{4+\frac{3\pi}{2}} 2 - \sqrt{x - \frac{3\pi}{2}}\, \mathrm{d}x - \int_{\frac{3\pi}{2}}^{2\pi} (1-\cos\theta)(1-\cos\theta)\, \mathrm{d}\theta$ $= \left[2x - \frac{\left(x - \frac{3\pi}{2}\right)^{\frac{3}{2}}}{\frac{3}{2}} \right]_{1+\frac{3\pi}{2}}^{4+\frac{3\pi}{2}} - \int_{\frac{3\pi}{2}}^{2\pi} (1-\cos\theta)^2\, \mathrm{d}\theta$ $= \left[2x - \frac{2\left(x - \frac{3\pi}{2}\right)^{\frac{3}{2}}}{3} \right]_{1+\frac{3\pi}{2}}^{4+\frac{3\pi}{2}} - \int_{\frac{3\pi}{2}}^{2\pi} (1 - 2\cos\theta + \cos^2\theta)\, \mathrm{d}\theta$ $= \left[2\left(4+\frac{3\pi}{2}\right) - \frac{2\left(4+\frac{3\pi}{2}-\frac{3\pi}{2}\right)^{\frac{3}{2}}}{3} - \left(2\left(1+\frac{3\pi}{2}\right) - \frac{2\left(1+\frac{3\pi}{2}-\frac{3\pi}{2}\right)^{\frac{3}{2}}}{3} \right) \right]$ $- \int_{\frac{3\pi}{2}}^{2\pi} (1 - 2\cos\theta + \frac{1}{2}(1+\cos 2\theta))\, \mathrm{d}\theta$ $= \frac{4}{3} - \left[\frac{3}{2}\theta - 2\sin\theta + \frac{1}{2}\left(\frac{\sin 2\theta}{2}\right) \right]_{\frac{3\pi}{2}}^{2\pi}$ $= \frac{4}{3} - \left[\frac{3}{2}(2\pi) - \left(\frac{3}{2}\left(\frac{3}{2}\pi\right) - 2(-1) \right) \right]$ $= \frac{4}{3} - \left(\frac{3\pi}{4} - 2 \right)$

	$=\dfrac{10}{3}-\dfrac{3\pi}{4}$
(iv)	Equation of normal to C at point with parameter α is $y-(1-\cos\alpha)=\left(-\tan\dfrac{\alpha}{2}\right)\left(x-(\alpha-\sin\alpha)\right)$. When normal crosses x-axis, $y=0$. $0-(1-\cos\alpha)=\left(-\tan\dfrac{\alpha}{2}\right)\left(x-(\alpha-\sin\alpha)\right)$ $x=\dfrac{\cos\alpha-1}{-\tan\dfrac{\alpha}{2}}+\alpha-\sin\alpha$ $x=\dfrac{\left(1-2\sin^2\dfrac{\alpha}{2}\right)-1}{-\dfrac{\sin\dfrac{\alpha}{2}}{\cos\dfrac{\alpha}{2}}}+\alpha-\sin\alpha$ $x=2\sin\dfrac{\alpha}{2}\cos\dfrac{\alpha}{2}+\alpha-\sin\alpha$ $x=\sin\alpha+\alpha-\sin\alpha=\alpha$ The normal to C at P crosses the x-axis at the point with coordinates $(\alpha,0)$. (Shown)