1 Two curves with equations $y=\sqrt{a}\,x+bx^2$ and $y=\ln(cx)$ intersect at the points where $x = 1$, $x = 2$ and $x = 3$. Find the values of a, b and c, correct to 3 decimal places. [4]

2 A sequence $u_1, u_2, u_3, ...$ is given by

$$u_1 = 2 \quad \text{and} \quad u_{n+1} = 4u_n + 2 \quad \text{for } n \geq 1.$$

Use the method of mathematical induction to prove that

$$u_n = \frac{2}{3}\left(4^n - 1\right).$$ [5]

3 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

The vectors $\mathbf{a}$ and $\mathbf{b}$ are given by

$$\mathbf{a} = 3p\mathbf{i} - 2p\mathbf{j} + 6p\mathbf{k} \quad \text{and} \quad \mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k},$$

where p is a positive constant. It is given that $\mathbf{a}$ is a unit vector.

(i) Find the exact value of p. [2]

(ii) Give a geometrical interpretation of $|\mathbf{a}\bullet\mathbf{b}|$. [1]

(iii) By expanding $(\mathbf{b}-\mathbf{a})\bullet(\mathbf{b}-\mathbf{a})$, find the exact value of $|\mathbf{b}|^2 + |\mathbf{a}|^2 - (\mathbf{b}-\mathbf{a})\bullet(\mathbf{b}-\mathbf{a})$. [3]

4 **(a)** The sequence of positive real numbers $u_1, u_2, u_3, ...$ is such that $u_{n+1} = 4 - \dfrac{5}{u_n + 2}$. By considering $u_{n+1} - u_n$, find algebraically the range of values of u_n for which the sequence is strictly increasing. [3]

(b) Given that $\mathrm{f}(r) = r(r+1)!$, simplify $\mathrm{f}(r) - \mathrm{f}(r-1)$. Hence find the sum of the first N terms of the series $2\times1! + 5\times2! + 10\times3! + ...$ in terms of N. [4]

5 **(i)** Use the substitution $u=\sqrt{x-1}$ to find $\int x\sqrt{x-1}\,\mathrm{d}x$. [4]

(ii) Hence, find the exact value of $\int_1^2 x^2\sqrt{x-1}\,\mathrm{d}x$. [4]

6 **(a)** Show that $\dfrac{1-\mathrm{i}\tan\theta}{1+\mathrm{i}\tan\theta}=\mathrm{e}^{-\mathrm{i}(2\theta)}$, where $-\dfrac{\pi}{2}<\theta<\dfrac{\pi}{2}$. [2]

(b) The complex number z is given by $z=1+2a\mathrm{i}$, where a is a non-zero real number. The complex number w is given by $w=r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$.

(i) Given that z^3 is real, find the possible values of z. [3]

(ii) For the value of z found in part **(i)** for which $a<0$, find θ such that $\arg\left(\dfrac{p^2}{w^4}\right)=\dfrac{\pi}{12}$, where $p=zw^*$. [3]

7 The functions f and g are defined by

$$\mathrm{f}: x \to -\mathrm{e}^{-x}-2, \qquad x\in\mathbb{R},\ x<5,$$

$$\mathrm{g}: x \to \frac{x}{h}+k, \qquad x\in\mathbb{R}, \text{ where } h \text{ and } k \text{ are positive constants.}$$

(i) Using a graphical method, explain why there are no real values of x which satisfy $\mathrm{f}(x)=\mathrm{f}^{-1}(x)$. [2]

(ii) State a sequence of transformations which transform the graph of $y=x$ to the graph of $y=\mathrm{g}(x)$. [2]

(iii) Find, in similar form, the composite function gf. [2]

(iv) Given that $h=\dfrac{3}{2}$, find the least integer k for which $\mathrm{gf}(x)=(\mathrm{gf})^{-1}(x)$ has at least one solution. [3]

8 The line l has equation $\frac{x+4}{-2}=\frac{y-4}{6}=\frac{z+1}{4}$, and the plane p has equation $x-3y-2z=0$.

(i) Show that l is perpendicular to p. [2]

(ii) Find the coordinates of the point of intersection of l and p. [4]

(iii) Show that the point A with coordinates $(-10, 22, 11)$ lies on l. Find the coordinates of the point B which is the mirror image of A in p. [3]

(iv) Find the area of triangle OAB, where O is the origin, giving your answer to the nearest whole number. [3]

9 A n metre by n metre piece of cardboard is used to make a box, where n is a positive constant. Four squares of side x metre and two rectangles of dimension x metre by y metre as shown in Figure 1 are cut off and the remaining cardboard is folded to form a closed box of height x metre as shown in Figure 2.

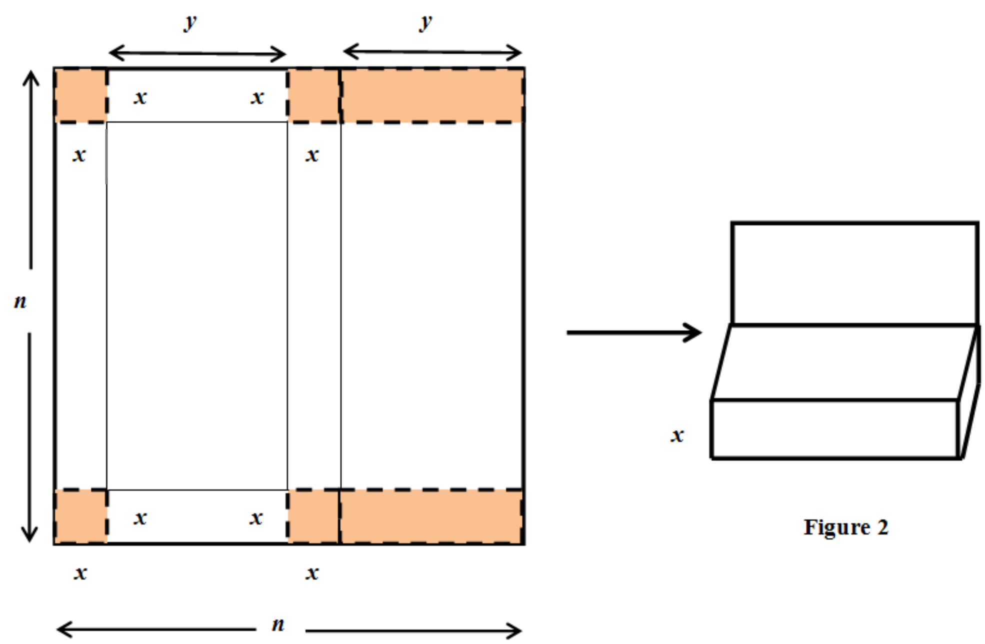


Figure 1

Figure 2

(i) Show that the volume V cubic metres of the box is given by

$$V=\frac{1}{2}x(n^2-4nx+4x^2).$$ [3]

(ii) Without using a calculator, find the value of x that gives a stationary value of V, and explain why there is only one answer. [6]

(iii) Hence find the maximum volume of the box in terms of n. [3]

10 **(i)** Given that $y=\tan^{-1}(2x)$, show that $\dfrac{d^2y}{dx^2}+4x\left(\dfrac{dy}{dx}\right)^2=0$. Hence, find the Maclaurin series for y up to and including the term in x^3. [5]

(ii) Using your answer in part **(i)**, find the Maclaurin series for $e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right]$ up to and including the term in x^3.

[You may use standard results given in the List of Formulae (MF15).] [3]

(iii) Use your answer in part **(ii)** to give an approximation for $\int_0^k e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right]dx$ in terms of k, and evaluate this approximation in the case where $k=0.8$. [3]

(iv) Use your calculator to find an accurate value for $\int_0^{0.8} e^{\frac{x}{2}}\left[\tan^{-1}(2x)\right]dx$. Why is the approximation in part **(iii)** not very good? [2]

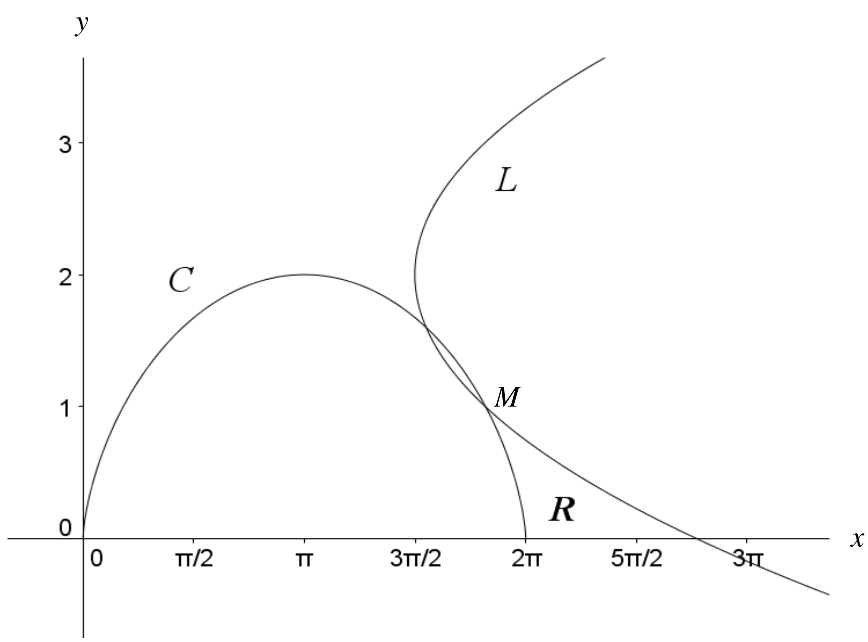
11 A curve C has parametric equations

$$x = \theta - \sin\theta, \qquad y = 1 - \cos\theta,$$

where $0 \le \theta \le 2\pi$.

(i) Show that $\dfrac{dy}{dx} = \cot\dfrac{1}{2}\theta$ and find the gradient of C at the point where $\theta = \pi$. What can be said about the tangents to C as $\theta \to 0$ and $\theta \to 2\pi$? [5]

(ii) It is given that the gradient of the tangent to C at the point M is -1. Show that $\theta = \dfrac{3\pi}{2}$ and find the exact coordinates of M. [3]



A curve L has equation $x = (y-2)^2 + \dfrac{3\pi}{2}$. The curves C and L intersect at the point M.

(iii) Find the exact value of the area of the region $\boldsymbol{R}$ bounded by C, L and the x-axis. [5]

(iv) A point P on C has parameter α, where $0 < \alpha < \dfrac{\pi}{2}$. Show that the normal to C at P crosses the x-axis at the point with coordinates $(\alpha, 0)$. [3]

~~~ End of paper ~~~