

Section A: Pure Mathematics [40 marks]

- 1** The complex number z satisfies the equation $|z - (9 - 4i)| = 5$.
- (i) Sketch an Argand diagram to illustrate this equation. [2]
 - (ii) Given that $|z|$ is as small as possible,
 - (a) find the exact value of $|z|$, [2]
 - (b) hence find an exact expression for z , in the form $x + iy$. [2]
 - (iii) It is given instead that $-\pi < \arg z \leq \pi$ and that $|\arg z|$ is as large as possible. Find the value of $\arg z$ in radians, correct to 4 significant figures. [2]
- 2** A curve C has equation $y = \frac{-x^2 + 4x + 12}{x + 3}$.
- (i) Using an algebraic method, find the range of values of y for which C does not exist. [3]
 - (ii) Sketch C , stating the exact coordinates of any points of intersection with the axes, the exact coordinates of turning points and the equations of any asymptotes. [3]
 - (iii) The region enclosed by C , the lines $x = -2$ and $y = 4$ is denoted by R . Find the numerical value of the volume of revolution when region R is rotated through 2π radians about the x -axis. [3]
- 3** On a certain large island, scientists have predicted that the number of rabbits will increase after a successful breeding program. This rate of increase is modelled by the differential equation
- $$\frac{dy}{dt} + y = 4t + 3,$$
- where y is the number of rabbits in millions and t is the time in decades.
- (i) Using the substitution $u = ye^t$, solve the differential equation given that there are initially 0.2 million rabbits. [5]
 - (ii) Hence, find the expected number of rabbits in a decade. [1]
- Alarmed by the above prediction, the scientists decide to introduce specially-bred foxes to regulate the number of rabbits. These foxes will reduce the number of rabbits by $4t$ million every decade.
- (iii) Write down the differential equation formed after introducing the specially-bred foxes. Hence, solve this new differential equation with the same initial condition of 0.2 million rabbits. [3]
 - (iv) What would happen to the number of rabbits in the long run? [1]

- 4 In order to pay for her university course fees, Jaslyn applies for a tuition fee loan and a study loan. The tuition fee loan covers 90% of her course fees while the study loan covers the remaining 10%. Repayment for both loans commences at the start of the month after graduation and interest is charged at the end of every month.

Type of loan	Mode of repayment	Interest rate on outstanding amount	Maximum repayment period
Tuition Fee Loan	Fixed monthly instalments, k (minimum of \$100)	0.4% per month	20 years
Study Loan	Fixed monthly instalments (minimum of \$100)	0%	5 years

Jaslyn's course fees add up to a total of \$28000. She intends to maximise her repayment period as much as possible. (Leave your answer to the nearest cent.)

- (i) How many months will it take for Jaslyn to fully repay her study loan? [2]
- (ii) Use the formula for the sum of a geometric progression to show that the value of the outstanding amount of her tuition fee loan at the end of n th month is

$$1.004^n (25200) - 251k (1.004^n - 1).$$

Hence find the minimum monthly instalment Jaslyn has to pay for her tuition fee loan. [6]

- (iii) What is the total interest Jaslyn has to pay for the tuition fee loan? [2]
- (iv) How much more will she have to pay if the interest rate on the tuition fee loan is revised to 0.5% per month instead? [3]

Section B: Statistics [60 marks]

- 5 The continuous random variable X has the distribution $N(\mu, \sigma^2)$. It is known that $P(2X < 1) = 0.3$ and $P(-\mu < X < 3\mu) = 0.96$. Calculate the values of μ and σ . [4]
- 6 A university has a population of 6400 students. The university is interested to find out the opinions of students from various faculties on a university wide initiative and intends to invite 200 students for a focus group discussion.
- (i) Explain how systematic sampling could be carried out to choose the 200 students. Explain briefly why this may not provide the representative sample that the university needs for its focus group discussion. [3]
- (ii) Name a more appropriate sampling method, and explain how it can be carried out to provide the representative sample that is needed by the university for a focus group discussion. [2]

- 7 In a certain country, the probability of more than 10 cm of snowfall on any particular winter day is 0.19.
- (i) Find the probability that, in a period of 7 winter days, fewer than 3 days have more than 10 cm of snowfall. [2]
- (ii) For any 12 randomly chosen 7 winter day periods, find the probability that exactly 4 of these periods have at least 2 days with more than 10 cm of snowfall. [3]
- 8 A set of 40 cards is made up of cards chosen from a number of packs of ordinary playing cards. The numbers of cards of each type are given in the following table.

	Spades	Hearts	Diamonds	Clubs
King	3	2	6	4
Queen	4	5	4	1
Jack	1	3	2	5

For example, there are 4 Kings of Clubs and 3 Jacks of Hearts. Three cards are taken from the set, at random and without replacement.

- (i) Find the probability that the cards taken are three Kings, three Queens or three Jacks. [3]
- (ii) Two Spades and one Diamond are taken. What is the probability that the cards taken are three Kings or three Queens or three Jacks? (For example, the three cards can consist of two Kings of Spades and one King of Diamonds.) [4]
- 9 The breaking strength of a climbing gear, x kN, is known to have mean 23 kN and standard deviation σ kN. It is decided that a new material will be added during its manufacture and a new batch of climbing gears is produced. The breaking strength of a random sample of 15 climbing gears from this new batch is measured. The results are summarised as follows:

$$\Sigma x = 371, \quad \Sigma x^2 = 9425.$$

- (i) Find the unbiased estimates of the population mean and variance. [2]
- (ii) Test, at the 10% significance level, whether the mean breaking strength of the climbing gear has increased, stating a necessary assumption. Explain why the Central Limit Theorem does not apply in this context. [6]

- 10** There are 10 spaniels, 14 retrievers and 6 poodles at a dog show. 7 dogs are selected to go through to the final.

(i) Find the number of different possible selections if there must be at least 1 spaniel, at least 2 retrievers and at least 3 poodles. [4]

2 spaniels, 2 retrievers and 3 poodles go through to the final.

(ii) The dogs are placed in a line. Find the number of different possible arrangements if the spaniels stand together and the retrievers stand together. [2]

(iii) The dogs are placed in a circle. Find the number of different possible arrangements if no two poodles are next to each other. [2]

- 11** A car is travelling along a stretch of road with speed v km/h when the brakes are applied. The car comes to rest after travelling a further distance of s metre. The values of s for 8 different values of v are given in the table, correct to 2 decimal places.

v	25	30	35	40	45	50	55	60
s	2.83	4.63	4.84	5.29	9.73	10.30	14.82	15.21

(i) Calculate the product moment correlation coefficient between v and \sqrt{s} . What does this indicate about the scatter diagram of the points (v, \sqrt{s}) ? [3]

(ii) It is given that the product moment correlation coefficient between v and s is 0.965, correct to 3 decimal places. State why the regression line of \sqrt{s} on v is more suitable than the regression line of s on v , and find the equation of the regression line of \sqrt{s} on v . [3]

(iii) Consider the equation of the regression line of \sqrt{s} on v . In the context of the question,

(a) comment on the value of the constant term, [2]

(b) interpret the slope of the regression line. [1]

(iv) Would you be willing to use this model to predict the further distance travelled if the speed is 70 km/h? Explain your answer with reason(s). [2]

- 12** The number of whales spotted per day during a cruise has the distribution $Po(1.2)$. The number of dolphins spotted per day during the same cruise has the distribution $Po(4.5)$. The two distributions are independent. Assume that the cruise covers the same location each day for all the tours.
- (i) Tom takes a 1-day cruise. Find the probability that he spots at least 3 whales. [2]
 - (ii) Jerry wants to go for an n -day cruise, where n is an integer. He wants to ensure that his chance of not spotting any whales is not more than 5%. Find the smallest possible value of n . [3]
 - (iii) Assume that the chance of spotting at least 20 whales or dolphins altogether in a 5-day cruise is 96%. In a particular year where there are 52 weeks, there is a 5-day cruise each week. Using a suitable approximation, find the probability that in that year, more than four 5-day cruises spotted less than 20 whales or dolphins altogether. [3]
 - (iv) Stella, a tour guide, leads one cruise per day for 100 consecutive days a year. Using a suitable approximation, find the probability that she spots at least four times as many dolphins as whales in a year. [4]

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