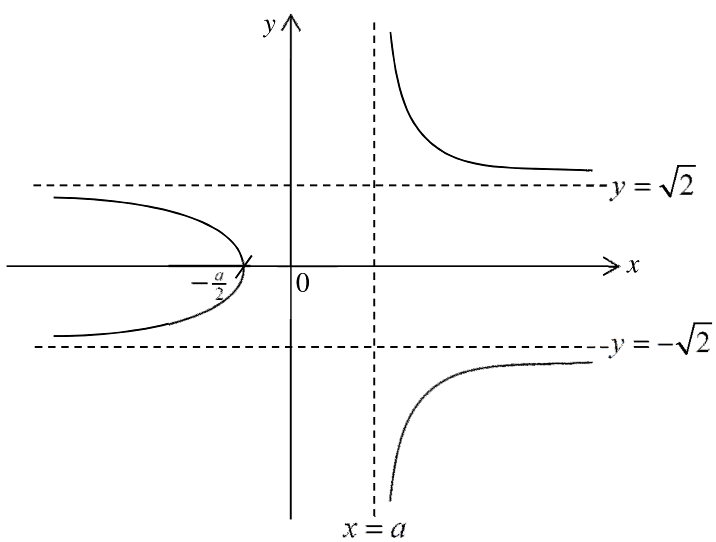
2014 H2 Math Prelim P1 Solutions

1(i)	$u_{n+1}=\dfrac{8}{u_n-4},\ n\geq 1$ As $n\to\infty$, $u_n\to l$ and $u_{n+1}\to l$ $l=\dfrac{8}{l-4}$ $l^2-4l=8$ $l^2-4l-8=0$ $l=\dfrac{4\pm\sqrt{4^2-4(-8)}}{2}$ $=2\pm 2\sqrt{3}$ Since the sequence comprises negative numbers, l cannot be positive. $\therefore l=2-2\sqrt{3}$	
1(ii)	From GC, the sequence is oscillating and converging to l.	
2(a)	$y=\dfrac{2x+a}{x-a}=\dfrac{2(x-a)+3a}{x-a}=2+\dfrac{3a}{x-a}$ $y=\dfrac{1}{x}\xrightarrow{\text{A}}y=\dfrac{1}{x-a}\xrightarrow{\text{B}}y=\dfrac{3a}{x-a}\xrightarrow{\text{C}}y=2+\dfrac{3a}{x-a}$ A: Translate by a units in the positive x-direction. B: Scale parallel to the y-axis by factor $3a$. C: Translate by 2 units in the positive y-direction.	
2(b)		

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<tr><td>3(i)</td><td>

$\mathrm{f}(x) = 2 + \ln(1 + \sin x)$

$\mathrm{f}'(x) = \dfrac{\cos x}{1 + \sin x}$

$(1 + \sin x)\mathrm{f}'(x) = \cos x$

$(1 + \sin x)\mathrm{f}''(x) + (\cos x)\mathrm{f}'(x) = -\sin x$

Substituting $x = 0$, we have

$\mathrm{f}(0) = 2 + \ln(1 + \sin 0) = 2,$

$\mathrm{f}'(0) = \dfrac{\cos 0}{1 + \sin 0} = 1,$

$(1 + \sin 0)\mathrm{f}''(0) + (\cos 0)(1) = -\sin 0$

$\mathrm{f}''(0) = -1$

$\mathrm{f}(0) = 2,\ \mathrm{f}'(0) = 1,\ \mathrm{f}''(0) = -1$

$\therefore\ \mathrm{f}(x) \approx 2 + x - \dfrac{1}{2}x^2$

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<tr><td>3(ii)</td><td>

$$\begin{aligned}\mathrm{g}(x) &= \left[2 + \ln(1 + \sin x)\right]^{\frac{1}{2}} \\ &= \left[2 + x - \frac{1}{2}x^2\right]^{\frac{1}{2}} \\ &= \sqrt{2}\left[1 + \left(\frac{x}{2} - \frac{x^2}{4}\right)\right]^{\frac{1}{2}} \\ &= \sqrt{2}\left[1 + \frac{1}{2}\left(\frac{x}{2} - \frac{x^2}{4}\right) + \frac{\frac{1}{2}\left(\frac{1}{2} - 1\right)}{2!}\left(\frac{x}{2} - \frac{x^2}{4}\right)^2 + \ldots\right] \\ &= \sqrt{2}\left[1 + \frac{x}{4} - \frac{x^2}{8} - \frac{x^2}{32} + \ldots\right] \\ &\approx \sqrt{2}\left(1 + \frac{1}{4}x - \frac{5}{32}x^2\right)\end{aligned}$$

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<tr><td>3(iii)</td><td>

Approximation for $\int_0^{\frac{\pi}{2}} \mathrm{g}(x)\,\mathrm{d}x$:

$$\begin{aligned}\int_0^{\frac{\pi}{2}} \mathrm{g}(x)\,\mathrm{d}x &\approx \int_0^{\frac{\pi}{2}} \sqrt{2}\left(1 + \frac{1}{4}x - \frac{5}{32}x^2\right)\mathrm{d}x \\ &= 2.37\ \text{(to 3 s.f.)}\end{aligned}$$

Accurate value for $\int_0^{\frac{\pi}{2}} \mathrm{g}(x)\,\mathrm{d}x = 2.47$ (to 3 s.f.)

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$$
\begin{aligned}
RHS &= (k+1)(k+2)-2(k+2)+1 \\
&= (k+2)(k-1)+1 \\
&= k^2+k-2+1 \\
&= k^2+k-1 \\
&= LHS
\end{aligned}
$$

$$\sum_{k=1}^{n}\frac{k^2+k-1}{(k+2)!}$$

$$=\sum_{k=1}^{n}\frac{(k+1)(k+2)-2(k+2)+1}{(k+2)!}$$

$$=\sum_{k=1}^{n}\left[\frac{1}{k!}-\frac{2}{(k+1)!}+\frac{1}{(k+2)!}\right]$$

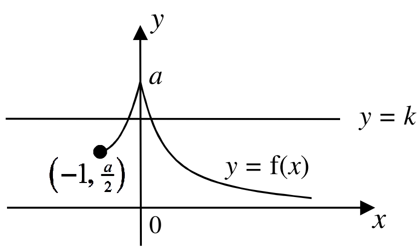
$$
\begin{array}{lccccc}
=[& \frac{1}{1!} & - & \frac{2}{2!} & + & \cancel{\frac{1}{3!}} \\
+ & \frac{1}{2!} & - & \cancel{\frac{2}{3!}} & + & \cancel{\frac{1}{4!}} \\
+ & \cancel{\frac{1}{3!}} & - & \cancel{\frac{2}{4!}} & + & \cancel{\frac{1}{5!}} \\
+ & \cancel{\frac{1}{4!}} & - & \cancel{\frac{2}{5!}} & + & \cancel{\frac{1}{6!}} \\
 & & & \vdots & & \\
+ & \cancel{\frac{1}{(n-2)!}} & - & \cancel{\frac{2}{(n-1)!}} & + & \cancel{\frac{1}{n!}} \\
+ & \cancel{\frac{1}{(n-1)!}} & - & \cancel{\frac{2}{n!}} & + & \frac{1}{(n+1)!} \\
+ & \cancel{\frac{1}{n!}} & - & \frac{2}{(n+1)!} & + & \frac{1}{(n+2)!} \quad]
\end{array}
$$

$$=\frac{1}{1!}-\frac{2}{2!}+\frac{1}{2!}+\frac{1}{(n+1)!}-\frac{2}{(n+1)!}+\frac{1}{(n+2)!}$$

$$=\frac{1}{2}-\frac{1}{(n+1)!}+\frac{1}{(n+2)!}$$

$$=\frac{1}{2}-\frac{n+2-1}{(n+2)!}$$

$$=\frac{1}{2}-\frac{n+1}{(n+2)!}$$

	$\sum_{k=4}^{n+3}\frac{(k-2)^2+k-3}{k!}=\sum_{r=2}^{n+1}\frac{r^2+(r+2)-3}{(r+2)!}$ $=\sum_{r=2}^{n+1}\frac{r^2+r-1}{(r+2)!}$ $=\frac{1}{2}-\frac{n+1+1}{(n+3)!}-\left[\frac{1}{2}-\frac{2}{3!}\right]$ $=\frac{1}{3}-\frac{n+2}{(n+3)!}$	
5(i)	 Since the line $y=k$, where $\frac{a}{2}\le k<a$, cuts the graph of $y=\mathrm{f}(x)$ at more than one point, $\therefore$ by the horizontal line test, f is not one-to-one. Therefore, f^{-1} does not exist.	
	Alternative: $\mathrm{f}(1)=\frac{a}{2},\ \mathrm{f}(-1)=\frac{a}{2},$ $\therefore\ \mathrm{f}(1)=\mathrm{f}(-1)$ but $1\ne-1$. Therefore f is not one-to-one and f^{-1} does not exist.	
5(ii)	Least $k=0$ Let $y=\frac{a}{x+1}$ since $x\ge 0$ $x=\frac{a}{y}-1$ $\mathrm{f}^{-1}:x\mapsto\frac{a}{x}-1,\ \ 0<x\le a\ \ \left(\because \mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_{\mathrm{f}}\right)$	
5(iii)	$\mathrm{f}^2(x)=x$ $\Rightarrow \mathrm{f}(x)=\mathrm{f}^{-1}(x),\quad x>0$ $\Rightarrow \mathrm{f}^{-1}(x)=x,\quad x>0,$ $\frac{a}{x}-1=x,\quad x>0$ $\Rightarrow x^2+x-a=0,\quad x>0,$ $\therefore x=\frac{-1+\sqrt{1+4a}}{2}$, (reject $x=\frac{-1-\sqrt{1+4a}}{2}<0$).	

Alternative:

$$\mathrm{f}^2(x) = x$$
$$\Rightarrow \mathrm{f}(x) = \mathrm{f}^{-1}(x), \quad x > 0$$
$$\Rightarrow \mathrm{f}(x) = x, \ x > 0,$$
$$\Rightarrow \frac{a}{|x|+1} = x, \ x > 0, \Rightarrow \frac{a}{x+1} = x (\because x > 0 \text{ and } a > 0)$$
$$\Rightarrow x^2 + x - a = 0$$
$$\therefore x = \frac{-1+\sqrt{1+4a}}{2}, (\text{reject } x = \frac{-1-\sqrt{1+4a}}{2} < 0).$$

Alternative:

$$\mathrm{f}^2(x) = x,$$
$$\frac{a}{\left|\frac{a}{|x|+1}\right|+1} = x \Rightarrow \frac{a(|x|+1)}{a+|x|+1} = x$$
$$\Rightarrow \frac{a(x+1)}{a+x+1} = x, \quad (\because a > 0 \ \& \ x > 0)$$
$$\Rightarrow x^2 + x - a = 0$$
$$\therefore x = \frac{-1+\sqrt{1+4a}}{2}, (\text{reject } x = \frac{-1-\sqrt{1+4a}}{2} < 0).$$

6(i)

In the empty region of the graph, the horizontal line $y = p$ does not intersect the curve at any points.

Consider $p = 2x + \dfrac{8k^2}{x-2}$:

$$p(x-2) = (2x)(x-2) + 8k^2$$
$$xp - 2p = 2x^2 - 4x + 8k^2$$
$$2x^2 - (4+p)x + 8k^2 + 2p = 0$$

$$\text{Discriminant, } b^2 - 4ac = (4+p)^2 - 4(2)(8k^2 + 2p)$$
$$= p^2 + 8p + 16 - 64k^2 - 16p$$
$$= p^2 - 8p + 16 - 64k^2$$

When $p^2 - 8p + 16 - 64k^2 = 0$,

$$p = \frac{8 \pm \sqrt{8^2 - 4(16 - 64k^2)}}{2} = 4 \pm 8k$$

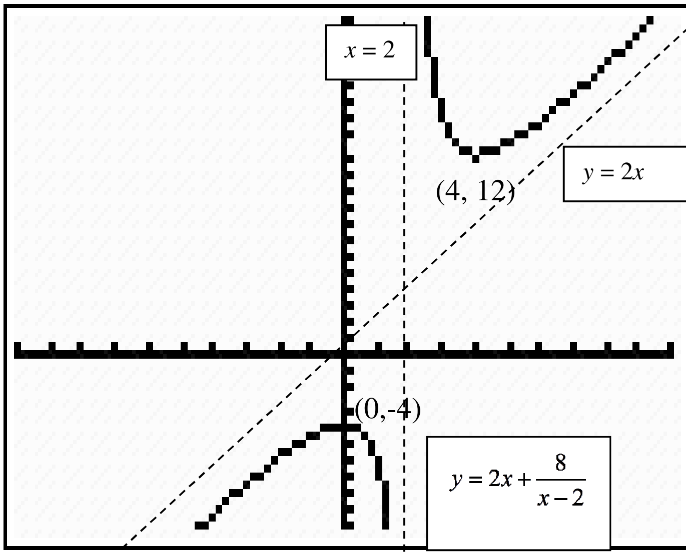
In empty region, curve does not intersect the line $y = p$, i.e. the equation $p = 2x + \dfrac{8k^2}{x-2}$ has no real solutions. This occurs when the discriminant is less than 0, i.e.

$p^2 - 8p + 16 - 64k^2 < 0$

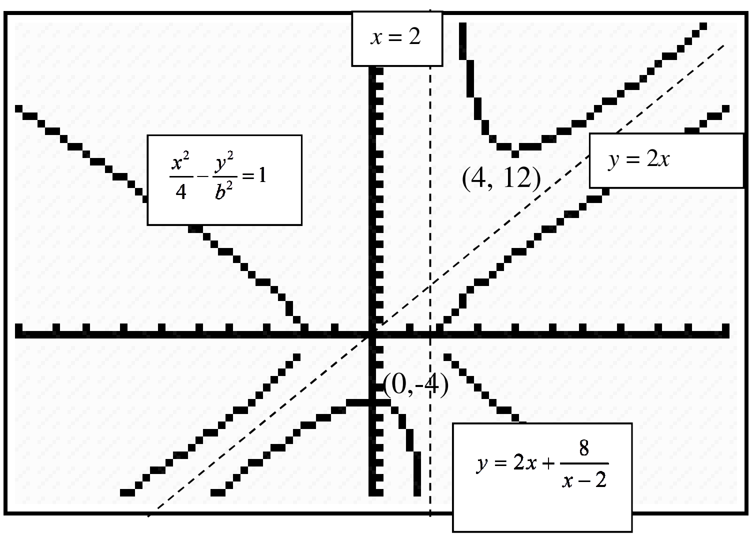
$4 - 8k < p < 4 + 8k$

Therefore y cannot lie between $4 - 8k$ and $4 + 8k$.

6(ii)



6(iii)



$b^2x^2 - 4y^2 = 4b^2 \quad \Rightarrow \quad \dfrac{x^2}{4} - \dfrac{y^2}{b^2} = 1$

This is a hyperbola with x-intercepts (-2, 0) and (2, 0) and asymptotes $y = \pm\dfrac{b}{2}x$.

For the hyperbola curve not to intersect curve C, the absolute value of the gradient of the asymptote of the hyperbola must not be greater than 2.

$\dfrac{b}{2} \le 2$ and $b > 0$

$\therefore \quad 0 < b \le 4$

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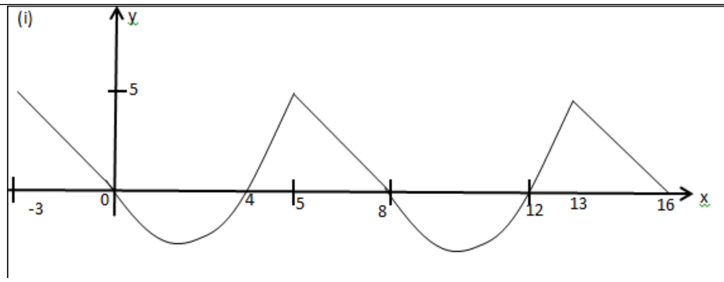
<tr><td>7(a)
(i)</td><td>
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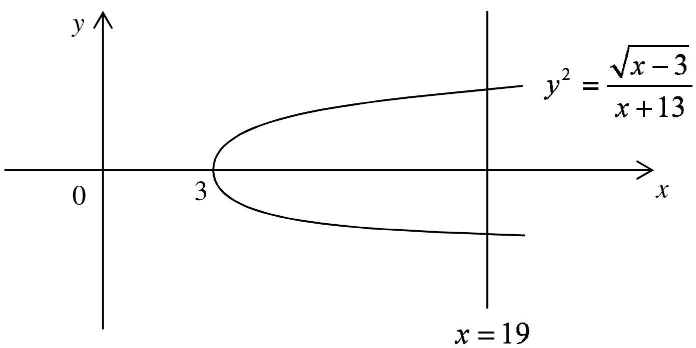
<tr><td>7(a)
(ii)</td><td>Area bounded by the curve and the x-axis between $x=-3$ and $x=13$ is

$$\int_{-3}^{0}\mathrm{f}(x)\ \mathrm{d}x-\int_{0}^{4}\mathrm{f}(x)\ \mathrm{d}x+\int_{4}^{5}\mathrm{f}(x)\ \mathrm{d}x$$
$$+\int_{5}^{8}\mathrm{f}(x)\ \mathrm{d}x-\int_{8}^{12}\mathrm{f}(x)\ \mathrm{d}x+\int_{12}^{13}\mathrm{f}(x)\ \mathrm{d}x$$
$$=2\int_{5}^{8}\mathrm{f}(x)\ \mathrm{d}x+2\int_{4}^{5}\mathrm{f}(x)\ \mathrm{d}x-2\int_{0}^{4}\mathrm{f}(x)\ \mathrm{d}x$$
$$=2\int_{5}^{8}\left(-\frac{5}{3}x+\frac{40}{3}\right)\ \mathrm{d}x+2\int_{4}^{5}\left((x-2)^2-4\right)\ \mathrm{d}x$$
$$-2\int_{0}^{4}\left((x-2)^2-4\right)\ \mathrm{d}x$$
$$=41\text{ units}^2$$</td><td></td></tr>
<tr><td>7(b)</td><td>

Volume required

$$=\pi\int_{3}^{19}\frac{\sqrt{x-3}}{x+13}\ \mathrm{d}x$$

$t=\sqrt{x-3}$
$t^2=x-3$
$x=t^2+3$
$\int \mathrm{d}x=\int 2t\ \mathrm{d}t$

$$=\pi\int_{0}^{4}\frac{t}{t^2+16}(2t)\ \mathrm{d}t$$
$$=\pi\int_{0}^{4}\frac{2t^2}{t^2+16}\ \mathrm{d}t$$
$$=\pi\int_{0}^{4}\left(2-\frac{32}{t^2+16}\right)\mathrm{d}t$$
$$=\pi\left[2t-\frac{32}{4}\tan^{-1}\left(\frac{t}{4}\right)\right]_0^4$$
$$=\pi\left[8-8\tan^{-1}\left(\frac{4}{4}\right)\right]=2\pi(4-\pi)\text{units}^3$$</td><td></td></tr>
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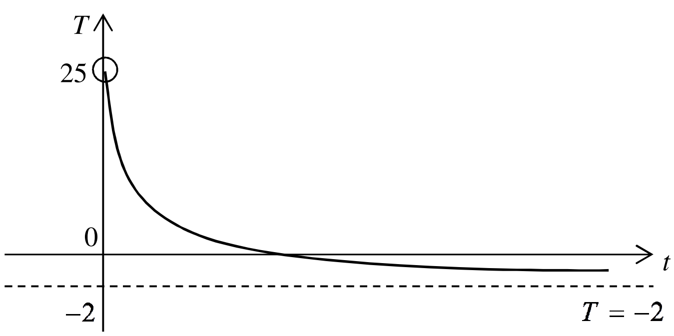
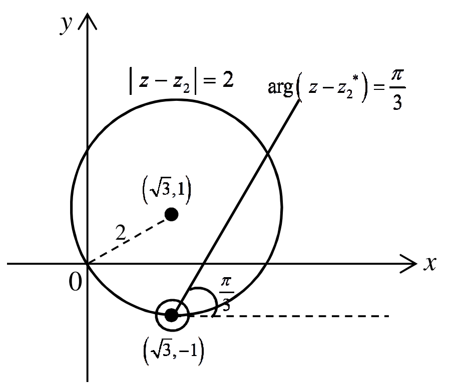
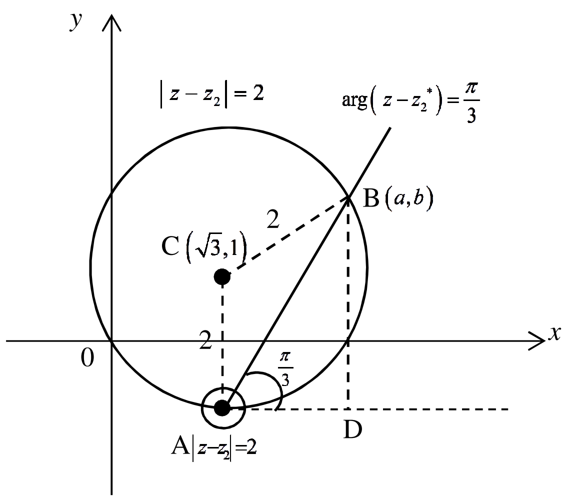
8(i)	Let T be the temperature of a body and temperature of the surroundings be T_0. $\frac{dT}{dt} = -m(T - T_0)$, $m > 0$	
8(ii)	$\int \frac{1}{T - T_0}\, dT = \int -m\, dt$ $\ln(T - T_0) = -mt + C$ $T = e^{-mt+C} + T_0$ $T = Ae^{-mt} + T_0$, where $A = e^C$ $\therefore\ T = Ae^{-kt} + T_0$, where $k = m$	
8(iii)	Given $T_0 = -2\ °C$ When $t = 0$, $T = 25\ °C$ $25 = A - 2 \quad \Rightarrow \quad \therefore\ A = 27$ $T = 27e^{-kt} - 2$ 	
8(iv)	Let x be the number of minutes passed after the heater has broken down. When $t = x$, $T = 18$. $18 = 27e^{-kx} - 2 \quad \Rightarrow \quad e^{-kx} = \frac{20}{27}$ --- (1) When $t = x + 30$, $T = 12$. $12 = 27e^{-k(x+30)} - 2 \ \Rightarrow\ e^{-k(x+30)} = \frac{14}{27}$ ---(2) $\frac{(1)}{(2)}$: $e^{30k} = \frac{20}{14} = \frac{10}{7}$ $\therefore\ k = \frac{1}{30}\ln\left(\frac{10}{7}\right)$ From equation (1), $e^{-(\frac{1}{30}\ln\frac{10}{7})x} = \frac{20}{27} \ \Rightarrow\ x = 25.24$ mins Therefore time of break down = 4.00 p.m. – 25 mins = 3.35 p.m.	

9(a) (i)	Shortest distance from point A to plane Π_1 $= \dfrac{\left\lvert \overrightarrow{OA} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} - 3 \right\rvert}{\left\lVert \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rVert}$ $= \dfrac{\left\lvert \begin{pmatrix}1\\2\\3\end{pmatrix} . \begin{pmatrix}1\\1\\1\end{pmatrix} - 3 \right\rvert}{\left\lVert \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rVert}$ $= \dfrac{3}{\sqrt{3}} = \sqrt{3}$ units	
9(a) (ii)	Acute angle between l and Π_1 $= \sin^{-1} \dfrac{\left\lvert \begin{pmatrix}2\\-3\\-2\end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rvert}{\left\lVert \begin{pmatrix}2\\-3\\-2\end{pmatrix} \right\rVert \left\lVert \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rVert}$ $= \sin^{-1}\left(\dfrac{3}{\sqrt{51}}\right)$ $= 24.83989 = 24.8°$	
	<u>Alternative:</u> Acute angle between l and Π_1 $= 90° - \cos^{-1} \dfrac{\left\lvert \begin{pmatrix}2\\-3\\-2\end{pmatrix} \cdot \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rvert}{\left\lVert \begin{pmatrix}2\\-3\\-2\end{pmatrix} \right\rVert \left\lVert \begin{pmatrix}1\\1\\1\end{pmatrix} \right\rVert}$ $= 90° - \cos^{-1}\left(\dfrac{3}{\sqrt{51}}\right)$ $= 24.83989 = 24.8°$	

9(a) (iii)	$\dfrac{\sqrt{3}}{AQ}=\sin\left(24.83989\right)$ $AQ=\dfrac{\sqrt{3}}{\sin\left(24.83989\right)}=4.12$ units	
9(b) (i)	$\overrightarrow{OC}=\left(1-\lambda\right)\mathbf{a}+\lambda\mathbf{b}$	
9(b) (ii)	Since $\overrightarrow{OC}\perp\overrightarrow{AB}$, $\overrightarrow{OC}\cdot\overrightarrow{AB}=0$. $\overrightarrow{OC}\cdot\overrightarrow{AB}$ $=\left[\left(1-\lambda\right)\mathbf{a}+\lambda\mathbf{b}\right]\cdot\left(\mathbf{b}-\mathbf{a}\right)$ $=\left(1-\lambda\right)\mathbf{a}\cdot\mathbf{b}-\left(1-\lambda\right)\mathbf{a}\cdot\mathbf{a}+\lambda\mathbf{b}\cdot\mathbf{b}-\lambda\mathbf{b}\cdot\mathbf{a}$ $=\left(1-2\lambda\right)\mathbf{a}\cdot\mathbf{b}-\left(1-\lambda\right)\lvert\mathbf{a}\rvert^2+\lambda\lvert\mathbf{b}\rvert^2$ $=\left(1-2\lambda\right)\mathbf{a}\cdot\mathbf{b}-\left(1-\lambda\right)\left(4\lvert\mathbf{b}\rvert^2\right)+\lambda\lvert\mathbf{b}\rvert^2$, since $\lvert\mathbf{a}\rvert=2\lvert\mathbf{b}\rvert$ $=\left(1-2\lambda\right)\mathbf{a}\cdot\mathbf{b}-\left(4-5\lambda\right)\lvert\mathbf{b}\rvert^2$ $\overrightarrow{OC}\cdot\overrightarrow{AB}=0$ $\left(1-2\lambda\right)\mathbf{a}\cdot\mathbf{b}-\left(4-5\lambda\right)\lvert\mathbf{b}\rvert^2=0$ $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{b}\rvert^2}=\dfrac{4-5\lambda}{1-2\lambda}$ $\dfrac{2\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}=\dfrac{4-5\lambda}{1-2\lambda}$ $\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}=\dfrac{4-5\lambda}{2\left(1-2\lambda\right)}$ $\therefore\quad\cos\theta=\dfrac{4-5\lambda}{2\left(1-2\lambda\right)}$, since $\cos\theta=\dfrac{\mathbf{a}\cdot\mathbf{b}}{\lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert}$	
9(b) (iii)	When ΔACO and ΔAOB are similar triangles, $\theta=90°$. So, $\cos\ \theta=\cos\ 90°=0$. Thus, $\dfrac{4-5\lambda}{2\left(1-2\lambda\right)}=0$ $\lambda=\dfrac{4}{5}$	
10(i)	$z_1^{\ 3}=\left(1+c\mathrm{i}\right)^3$ $=1+3c\mathrm{i}+3\left(c\mathrm{i}\right)^2+\left(c\mathrm{i}\right)^3$ $=1-3c^2+\mathrm{i}\left(3c-c^3\right)$	

10(ii)	Since $1+c\mathrm{i}$ is a root of the equation $z^3-7z^2+kz-15=0$, $(1+c\mathrm{i})^3-7(1+c\mathrm{i})^2+k(1+c\mathrm{i})-15=0$ $1-3c^2+\mathrm{i}(3c-c^3)-7(1+2c\mathrm{i}-c^2)+k+kc\mathrm{i}-15=0$ $(1-3c^2-7+7c^2+k-15)+\mathrm{i}(3c-c^3-14c+kc)=0$ $(k-21+4c^2)+\mathrm{i}(kc-11c-c^3)=0$ Comparing real and imaginary parts, $k-21+4c^2=0$ $kc-11c-c^3=0$ Solve $k+4c^2=21$ $k-c^2=11$ (since $c>0$) We get: $c=\sqrt{2}$ (Since c is a positive real number, reject $-\sqrt{2}$) $k=13$	
10 (iii)	By GC, roots of equation are $1+\mathrm{i}\sqrt{2}$, $1-\mathrm{i}\sqrt{2}$ and 5.	
10 (iv)		
10 (v)		

	$\measuredangle ACB = \frac{2\pi}{3}$ $AB^2 = AC^2 + BC^2 - 2(AC)(BC)\cos A\hat{C}B$ $= 2^2 + 2^2 - 2(2)(2)\cos\left(\frac{2\pi}{3}\right)$ $= 12$ $AB = \sqrt{12} = 2\sqrt{3}$ $\frac{AD}{AB} = \cos\left(\frac{\pi}{3}\right) \Rightarrow AD = \left(2\sqrt{3}\right)\left(\frac{1}{2}\right) = \sqrt{3}$ $\frac{DB}{AB} = \sin\left(\frac{\pi}{3}\right) \Rightarrow DB = \left(2\sqrt{3}\right)\left(\frac{\sqrt{3}}{2}\right) = 3$ $a = \sqrt{3} + \sqrt{3} = 2\sqrt{3}$ $b = -1 + 3 = 2$ $\therefore$ Complex number that satisfies both loci is $2\sqrt{3} + 2\mathrm{i}$.	
11(i)	$p + q = 3$ $p + 2q = 4$ From GC, $p = 2, q = 1$	
11(ii)	$\begin{pmatrix}1\\1\\1\end{pmatrix} \times \begin{pmatrix}1\\k\\2\end{pmatrix} = \begin{pmatrix}2-k\\-1\\k-1\end{pmatrix}$ $l: \boldsymbol{r} = \begin{pmatrix}2\\0\\1\end{pmatrix} + \lambda\begin{pmatrix}2-k\\-1\\k-1\end{pmatrix}, \lambda \in \mathbb{R}$	
11 (iii)	if the three planes intersect each other at a point, then l is NOT parallel to Π_3 Therefore, $\begin{pmatrix}1\\\beta\\3\end{pmatrix}$ is NOT perpendicular to $\begin{pmatrix}2-k\\-1\\k-1\end{pmatrix}$ So $\begin{pmatrix}1\\\beta\\3\end{pmatrix} \cdot \begin{pmatrix}2-k\\-1\\k-1\end{pmatrix} \neq 0$ $\beta \neq 2k - 1$	
11 (iv)	if the three planes intersect each other at a line, then l is parallel to Π_3	

	Therefore, $\begin{pmatrix}1\\ \beta\\ 3\end{pmatrix}$ is perpendicular to $\begin{pmatrix}2-k\\ -1\\ k-1\end{pmatrix}$ So $$\begin{pmatrix}1\\ \beta\\ 3\end{pmatrix}\cdot\begin{pmatrix}2-k\\ -1\\ k-1\end{pmatrix}=0$$ $$\beta = 2k-1$$ **<u>AND</u>** $(2,0,1)$ is on Π_3 So $$2+3=\mu$$ $$\mu = 5$$	
11 (v)	CASE 1: IF $\beta \neq 2k-1$ since $k=\beta$ $\qquad k \neq 2k-1$ Hence, for $k=\beta\neq 1$, the three planes will intersect each other at a point. CASE 2: IF $\beta = 2k-1$ Then, since $k=\beta$ Solving, we get $k=\beta=1$ Therefore, $\mu = 5$ So, the three planes will intersect each other in a line. Therefore, the three planes will always have at least one common point.	
12(a)	Let r be the radius of the circle of petrol, A be the area of the circle of petrol, V be the volume of petrol that leaks out. Given: $\frac{dV}{dt}=0.0084\ \text{m}^3\text{s}^{-1}$ and $V=0.002A$ Hence, $\frac{dV}{dt}=0.002\frac{dA}{dt} \Rightarrow \frac{dA}{dt}=\frac{0.0084}{0.002}=4.2\ \text{m}^2\text{s}^{-1}$ But $A=\pi r^2 \Rightarrow \frac{dA}{dt}=2\pi r\frac{dr}{dt}$ When the radius $r=3$ m, $\frac{dr}{dt}=\frac{4.2}{2\pi(3)}=0.22\ \text{ms}^{-1}$(to 2 decimal places) Therefore the radius is increasing at a rate of $0.22\ \text{ms}^{-1}$.	

12(b) (i)	The height of the trapezium is $h=\sqrt{x^2-\left(\frac{x}{2}\right)^2}=\sqrt{\frac{3}{4}x^2}=\frac{\sqrt{3}}{2}x$ $V=\frac{1}{2}\left[(a-2x)+a-x\right]\left(\frac{\sqrt{3}}{2}x\right)\left(\frac{2}{\sqrt{3}}a-2\left(\frac{\sqrt{3}}{2}x\right)\right)$ $=\frac{\sqrt{3}}{2}x\left(a-\frac{3}{2}x\right)\left(\frac{2}{\sqrt{3}}a-\sqrt{3}x\right)$ $=x\left(a-\frac{3}{2}x\right)\left(a-\frac{3}{2}x\right)$ $=x\left(a-\frac{3}{2}x\right)^2$	
12(b) (ii)	$V=x\left(a-\frac{3}{2}x\right)^2$ $\frac{\mathrm{d}V}{\mathrm{d}x}=x\left(2\left(a-\frac{3}{2}x\right)\left(-\frac{3}{2}\right)\right)+\left(a-\frac{3}{2}x\right)^2$ $=\left(a-\frac{3}{2}x\right)\left(-3x+a-\frac{3}{2}x\right)$ $=\left(a-\frac{3}{2}x\right)\left(a-\frac{9}{2}x\right)$ When $\frac{\mathrm{d}V}{\mathrm{d}x}=0$, $\left(a-\frac{3}{2}x\right)\left(a-\frac{9}{2}x\right)=0 \quad \Rightarrow \quad x=\frac{2}{3}a \quad$ or $\quad x=\frac{2}{9}a$ Since the width of the container will be $\left(\frac{2}{\sqrt{3}}a-\sqrt{3}\left(\frac{2}{3}a\right)\right)=0$ when $x=\frac{2}{3}a$, we reject $x=\frac{2}{3}a$. $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=\left(a-\frac{3}{2}x\right)\left(-\frac{9}{2}\right)+\left(a-\frac{9}{2}x\right)\left(-\frac{3}{2}\right)$ When $x=\frac{2}{9}a$, $\frac{\mathrm{d}^2V}{\mathrm{d}x^2}=\left(a-\frac{3}{2}\left(\frac{2}{9}a\right)\right)\left(-\frac{9}{2}\right)+\left(a-\frac{9}{2}\left(\frac{2}{9}a\right)\right)\left(-\frac{3}{2}\right)$ $=\left(a-\frac{1}{3}a\right)\left(-\frac{9}{2}\right)+0$ $=-3a<0$ Hence at $x=\frac{2}{9}a$, V is maximum.	

	Maximum $V = \left(\frac{2}{9}a\right)\left(a - \frac{3}{2}\left(\frac{2}{9}a\right)\right)^2 = \left(\frac{2}{9}a\right)\left(\frac{2}{3}a\right)^2 = \frac{8}{81}a^3$.	