NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION SOLUTIONS

Higher 2

MATHEMATICS **9740/02**

Paper 2 **17th September 2014**

3 Hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1 By definition, $\mathbf{n} = \mathbf{l} \times \mathbf{m} \Rightarrow \mathbf{n} \perp \mathbf{l}$ and $\mathbf{n} \perp \mathbf{m}$. Therefore, $\mathbf{l}.\mathbf{n} = 0$.

Let the foot of perpendicular from *C* to plane *OAB* be *C'*.

$\overrightarrow{CC'} // \mathbf{a} \times \mathbf{b} \ \Rightarrow \overrightarrow{CC'} = t\left(\mathbf{a} \times \mathbf{b}\right)$ for some scalar t

$$\overrightarrow{OC'} = \overrightarrow{OC} + \overrightarrow{CC'}$$

$\lambda \mathbf{a} + \mu \mathbf{b} = \mathbf{c} + t(\mathbf{a} \times \mathbf{b})$

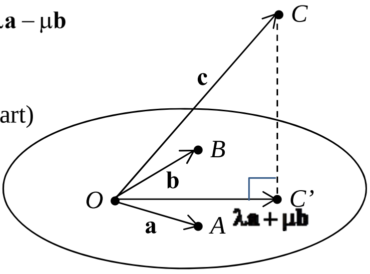
Therefore, there is a scalar such that $t(\mathbf{a} \times \mathbf{b}) = \mathbf{c} - \lambda \mathbf{a} - \mu \mathbf{b}$

Multiplying both sides by **a**,

$\mathbf{a} \cdot t(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \cdot (\mathbf{c} - \lambda \mathbf{a} - \mu \mathbf{b})$

$\Rightarrow 0 = \mathbf{a} \cdot \mathbf{c} - \lambda \mathbf{a} \cdot \mathbf{a} - \mu \mathbf{b} \cdot \mathbf{a}$ (using result from first part)

$\Rightarrow (\mathbf{a} \cdot \mathbf{a})\lambda + (\mathbf{a} \cdot \mathbf{b})\mu = \mathbf{a} \cdot \mathbf{c}$



2

Since $R_h = (0,\infty)$, $D_g = \mathbb{R}$

$R_h \subseteq D_g$, gh exists.

$$\mathrm{gh}(x) = \mathrm{g}\left[\ln(x-2)\right]$$

$$= e^{\ln(x-2)}$$

$$\mathrm{gh} : x \mapsto x-2, \quad x \in \mathbb{R}, \quad x > 3$$

$R_{gh} = (1,\infty)$

$\mathrm{f}(x+a) > \mathrm{gh}(x+a)$

$(x+a-a)(x+a-2) > x+a-2$

$(x)(x+a-2) - (x+a-2) > 0$

$(x+a-2)(x-1) > 0$

$x < 1$ or $x > 2-a$

Since $D_{gh} = (3,\infty)$, $x+a > 3 \Rightarrow x > 3-a$, hence range of values of x is $(3-a,\infty)$.

<u>Alternative solution</u>:

$\mathrm{f}(x) > \mathrm{gh}(x)$

$(x-a)(x-2) > x-2$

$(x-a)(x-2) - (x-2) > 0$

$(x-2)(x-a-1) > 0$

$x < 1+a$ or $x > 2$

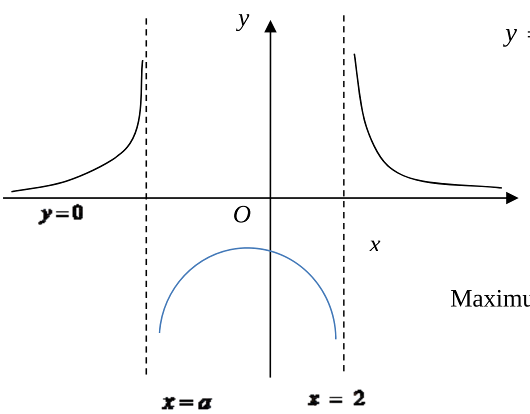
Replacing x by $x+a$,

$x+a < 1+a$ or $x+a > 2$

$x < 1$ or $x > 2-a$

Since $D_{gh} = (3,\infty)$, hence range of values of x is $(3,\infty)$.

Range of values of x for **(ii)** is $(3-a,\infty)$.



$$y = \frac{1}{\mathrm{f}(x)}$$

Maximum point $\left(\frac{2+a}{2}, -\frac{4}{(a-2)^2}\right)$

3

$$y=\frac{\ln\sqrt{1+x}}{1+x}$$

$$y=\frac{1}{2}\frac{\ln(1+x)}{1+x}$$

$$2(1+x)y=\ln(1+x)$$

$$2(1+x)\frac{\mathrm{dy}}{\mathrm{dx}}+2y=\frac{1}{1+x}$$

$$2(1+x)\frac{\mathrm{d^2y}}{\mathrm{dx^2}}+4\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{-1}{(1+x)^2}$$

$$2(1+x)\frac{\mathrm{d^3y}}{\mathrm{dx^3}}+6\frac{\mathrm{d^2y}}{\mathrm{dx^2}}=\frac{2}{(1+x)^3}$$

(i) When $x=0$, $y=0$, $\frac{dy}{dx}=\frac{1}{2}$, $\frac{d^2y}{dx^2}=-\frac{3}{2}$, $\frac{d^3y}{dx^3}=\frac{11}{2}$

$$\frac{\ln\sqrt{1+x}}{1+x}=0+x\left(\frac{1}{2}\right)+\frac{x^2}{2}\left(-\frac{3}{2}\right)+\frac{x^3}{6}\left(\frac{11}{2}\right)+\ldots$$

$$=\frac{1}{2}x-\frac{3x^2}{4}+\frac{11x^3}{12}+\ldots$$

(ii) $\frac{\ln\sqrt{1+x}}{1+x}=\frac{1}{2}[\ln(1+x)](1+x)^{-1}$

$$=\frac{1}{2}[x-\frac{x^2}{2}+\frac{x^3}{3}+\ldots](1-x+x^2-x^3+\ldots)$$

$$=\frac{1}{2}[x-x^2+x^3-\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^3}{3}+\ldots]$$

$$=\frac{1}{2}x-\frac{3x^2}{4}+\frac{11x^3}{12}+\ldots$$

(iii) $\int_0^{\frac{1}{4}\pi} y\ \mathrm{d}x=\int_0^{\frac{1}{4}\pi}\left(\frac{1}{2}x-\frac{3x^2}{4}+\frac{11x^3}{12}+\ldots\right)\mathrm{d}x$

$$=0.120$$

Approximation is not good as $x=\frac{\pi}{4}$ is not close to 0.

(iv) $y=\frac{1}{2}x$

4 **(a)(i)** The first three triangular numbers are 1, 3, 6.

(ii) $t_{r+1} - t_r = r + 1 \Rightarrow \sum_{r=1}^{n-1} (t_{r+1} - t_r) = \sum_{r=1}^{n-1} (r+1)$

$$\Rightarrow \begin{aligned} & t_2 - t_1 \\ & + t_3 - t_2 \\ & \vdots \\ & + t_n - t_{n-1} \end{aligned} = \sum_{r=1}^{n-1} (r+1)$$

$$\Rightarrow t_n - t_1 = \sum_{r=1}^{n-1} (r+1)$$

$$\Rightarrow t_n = 1 + 2 + 3 + \ldots + n = \frac{1}{2} n(n+1)$$

(iii) Sum of the first N triangular numbers

$$= \sum_{n=1}^{N} t_n = \frac{1}{2} \sum_{n=1}^{N} \left[n(n+1) \right] = \frac{1}{2} \left(\sum_{n=1}^{N} n^2 + \sum_{n=1}^{N} n \right)$$

$$= \frac{1}{2} \left[\frac{N}{6} (N+1)(2N+1) + \frac{N}{2} (N+1) \right]$$

$$= \frac{N}{6} (N+1)(N+2)$$

(i) $a_n > 2014 \Rightarrow 3^n + n - 1 > 2014$.

n	$3^n + n - 1$
6	734
7	2193
8	6568

By GC, least $n = 7$.
So the population of the bacteria first exceeds 2014 thousands 7 days after the start of the experiment.

(ii) Rate of growth of H2 Bacteria 2 days after the start of the experiment =

$$\left. \frac{\mathrm{d}a_n}{\mathrm{d}n} \right|_{n=2} = \left. \left(3^n \ln 3 + 1 \right) \right|_{n=2} = 9 \ln 3 + 1 \text{ thousands/day.}$$

or by GC, $\left. \frac{\mathrm{d}a_n}{\mathrm{d}n} \right|_{n=2} \approx 10.9$ thousands/day.

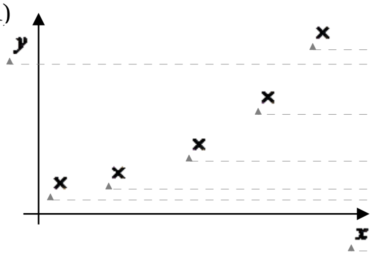
Let b_n denote the number of H3 Bacteria (in thousands) exactly n days after the start of the experiment. Then $\{b_n\}$ is an arithmetic sequence with first term 900 and common difference 800.
Therefore $b_n = 900 + (n-1)(800) = 800n + 100$.
We want least n such that $a_n - b_n > 0 \Rightarrow 3^n + n - 1 - (800n + 100) > 0 \Rightarrow 3^n - 799n - 101 > 0$.

	<table><tr><td>n</td><td>$3^n - 799n - 101$</td></tr><tr><td>7</td><td>–3507</td></tr><tr><td>8</td><td>68</td></tr><tr><td>9</td><td>12391</td></tr></table> By GC, the population of H2 Bacteria will exceed that of H3 Bacteria by the 8th day.
5	Numbered all 880 students names from 1 to 880. Chose a random starting number from 1 to 11th (as 880/80 = 11). Subsequently take every 11th name after the starting name. The process continues until 80 students have been chosen. The 80 students chosen may not have the correction proportion of male and female students as the list of students may not have an even spread of male and female students in the list. Stratified Sampling
6	probability that A wins the match = $0.6p + 0.6(1-p)(0.3) + 0.4(0.3)p$ = 0.18 + 0.54p P(B won the first set | Player B loses the match) = $\frac{0.4(0.3)p}{0.18+0.54p}$ $\frac{0.4(0.3)p}{0.18+0.54p} = 0.15$ Solving, p = 0.692 or $\frac{9}{13}$ When p = 0.65, P(A wins) = 0.531 P(3rd win in nth match) = ${}^{n-1}C_2 (0.531)^2 (1-0.531)^{n-3} \times (0.531) = 0.10866$ $\frac{(n-1)(n-2)}{2}(0.531)^3 (0.469)^{n-3} = 0.10866$ From GC, n = 7
7	P(Wise misses the bus) = P(X – Y < 0) $X - Y \sim N(25-15, 9+4)$ i.e. $X - Y \sim N(10,13)$ Thus, probability required = 0.0027728 ≈ 0.00277 (3 s.f.) T = 45 –（W + X）

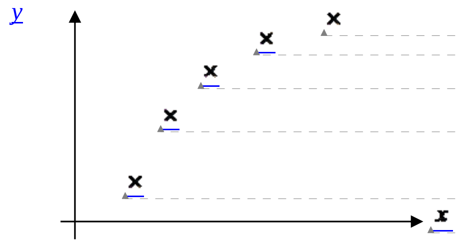
E(T) = 45 – 30 – 25 = – 10
Var(T) = 3 + 9 = 12

P(the bus arriving after 8.30 a.m.)
= P(T < 0)
= 0.99805 ≈ 0.998 (3 s.f.)

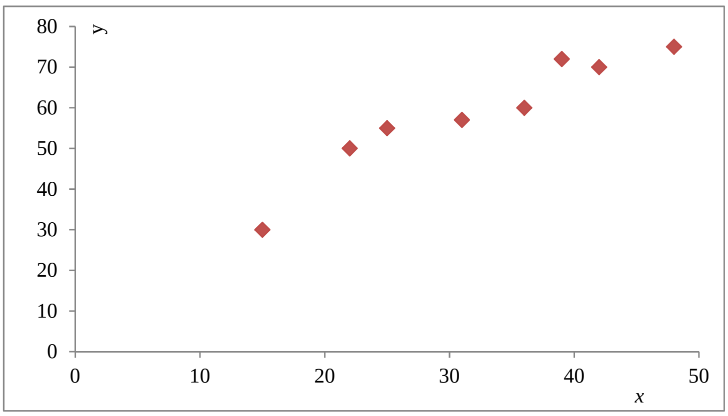
8 (A)



(B)



(ii)



(iii) From the diagram in (ii), as x increases, y increases in decreasing amount.
∴ (B) is the appropriate for modelling these values.

$r = -0.972$

$y = -935\frac{1}{x} + 91.6$

	(iv) when $y = 64 \Rightarrow x = 33.8$ As $y = 64$ falls within the given data range, the estimated value is valid. Neither regression line is suitable to estimate x since x is the independent variable which can be controlled.
9	Number of ways = 7! + 7! – 6! = 9360 Number of ways = (11-1)! x 2 / 4! x 12 = 3628800 Number of ways = (10–1)!/4! x 10 x 12 = 1814400
10	(a)(i) $H_0 : \mu = 40$ $H_1 : \mu > 40$ at 5% level of significance necessary assumption: the length of steel pins follows a normal distribution Apply t-test reject H_0 if $p-$ value < 0.05 $\bar{x} = 40 + \frac{3.7}{8} = 40.4625$ $s^2 = \frac{1}{7}[7.27 - \frac{(3.7)^2}{8}] = 0.794107$ $p-value = 0.09278$ Since $p-value$ >0.05, we do not reject H_0. There is insufficient evidence at 5% level of significance to conclude that mean length of steel pins is more than 40 mm. (ii) For a two-tail test, $p-value = 2(0.09278) > 0.05$. The conclusion is the same as (i) **(b)** $H_0 : \mu = k$ $H_1 : \mu \neq k$ at 4% level of significance Given $\bar{x}$ = 148.4 $s^2 = \frac{1}{49}(2372) = 48.40816$ For H_0 to be rejected, p-value < 0.04 if $\bar{x} < k$ or if $\bar{x} > k$

	$2P(\overline{X} < \overline{x}) < 0.04$ $P(\overline{X} < \overline{x}) < 0.02$ $P\left(Z < \dfrac{148.4-k}{\sqrt{\dfrac{48.408}{50}}}\right) < 0.02$ $\dfrac{148.4-k}{\sqrt{\dfrac{48.408}{50}}} < -2.05375$ $k > 150.4$ or $k < 146.4$ $2P(\overline{X} > \overline{x}) < 0.04$ $1-P(\overline{X} < \overline{x}) < 0.02$ $P(\overline{X} < \overline{x}) > 0.98$ $P\left(Z < \dfrac{148.4-k}{\sqrt{\dfrac{48.408}{50}}}\right) > 0.98$ $\dfrac{148.4-k}{\sqrt{\dfrac{48.408}{50}}} > 2.05375$
11	(a) Let X denote the demand for GreatRun tyres in a randomly chosen month.

$X \sim \text{Po}(4)$

Required prob. = $\text{P}(X \le 5) = \underline{0.78513}03874 = 0.785$ (3 s.f.)

(b) Using GC,

No. of tyres sold, x	Prob.
0	$\text{P}(X = 0) = 0.01832$
1	$\text{P}(X = 1) = 0.07326$
2	$\text{P}(X = 2) = 0.14653$
3	$\text{P}(X = 3) = 0.19537$
4	$\text{P}(X = 4) = 0.19537$
5	$\text{P}(X \ge 5) = 0.37116$

Hence, the most probable number of tyres sold is 5.

(c) Let the number of tyres the garage should keep in stock be N.

M1: Forming the inequality & use GC

$\text{P}(X > N) < 0.001$

$1 - \text{P}(X \le N) < 0.001$

Using GC,

N	$\text{P}(X > N)$
10	$0.00284 > 0.001$
11	$0.000915 < 0.001$

A1

Hence the least number of tyres the garage should keep in stock at the beginning of the month is 11.

(d) Let Y denote the demand for GreatRun tyres in a randomly chosen week.

M1: Calculate the probability of selling more than 1 tyre

$Y \sim \text{Po}(1)$

$\text{P}(Y > 1) = 1 - \text{P}(Y \le 1) = \underline{0.26424}11176$

B1: Define & identify the distribution

Let W denote the number of weeks where more than 1 GreatRun tyre is sold out of 4 weeks.

$W \sim \text{B}(4, 0.26424)$

A1

Required prob. = $\text{P}(W > 2) = 1 - \text{P}(W \le 2) = 0.059174 = 0.0592$ (3 s.f.)

B1: Define & identify the distribution
M1: Checking conditions
M1: Approximated distribution

M1: Conditional probability identified

M1: Apply continuity

(e) Let A denote the number of months the garage was not able to meet monthly demands out of 120 months.

$A \sim \text{B}(120, 1 - 0.78513)$ i.e. $A \sim \text{B}(120, 0.21487)$

Since $n = 120$ is large, $np = 25.7844 > 5$, $nq = 20.24410597 > 5$,

$A \sim \text{N}(25.7844, \underline{20.244}10597)$ i.e. $A \sim \text{N}(25.784, 20.244)$ approximately

Required prob. = $\text{P}\left(A \geq 36 \mid A > 12\right)$

$$= \frac{\text{P}\left(A \geq 36 \cap A > 12\right)}{\text{P}\left(A > 12\right)}$$

$$= \frac{\text{P}\left(A \geq 36\right)}{\text{P}\left(A > 12\right)}$$

$$= \frac{\text{P}\left(A > 35.5\right)}{\text{P}\left(A > 12.5\right)} \qquad \text{by continuity correction}$$

$$= 0.015432$$

$$= 0.0154 \text{ (3 s.f.)}$$

———— END OF PAPER ————