NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION SOLUTIONS

Higher 2

MATHEMATICS **9740/01**

Paper 1 **16th September 2014**

3 Hours

Additional Materials: Cover Sheet
Answer Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

1	$k=-1$ Let $y=2-(x+1)^2$ $x+1=\pm\sqrt{2-y}$ $x=-1+\sqrt{2-y}$ (NA) or $x=-1-\sqrt{2-y}$ ($\because x\le -1$) $\mathrm{f}^{-1}: x\mapsto -1-\sqrt{2-x}$, $x\le 2$ $\mathrm{f}\left(x\right)=\mathrm{f}^{-1}\left(x\right)$ $\Rightarrow\ \mathrm{f}\left(x\right)=x$ $\Rightarrow\ 2-\left(x+1\right)^2=x$ $\Rightarrow\ x^2+3x-1=0$ Using GC, $x=-3.303$ (since domain of f is $x\le -1$)
2	Area of A = $\int_0^{\frac{\pi}{2}} \mathrm{e}^x \sin x\,\mathrm{d}x$ $=\left[\mathrm{e}^x\sin x\right]_0^{\frac{\pi}{2}}-\int_0^{\frac{\pi}{2}}\mathrm{e}^x\cos x\,\mathrm{d}x$ $=\mathrm{e}^{\frac{\pi}{2}}-\left\{\left[\mathrm{e}^x\cos x\right]_0^{\frac{\pi}{2}}+\int_0^{\frac{\pi}{2}}\mathrm{e}^x\sin x\,\mathrm{d}x\right\}$ $=\mathrm{e}^{\frac{\pi}{2}}-\left\{-1+\int_0^{\frac{\pi}{2}}\mathrm{e}^x\sin x\,\mathrm{d}x\right\}$ $2\int_0^{\frac{\pi}{2}}\mathrm{e}^x\sin x\,\mathrm{d}x=\mathrm{e}^{\frac{\pi}{2}}+1$ $\int_0^{\frac{\pi}{2}}\mathrm{e}^x\sin x\,\mathrm{d}x=\frac{1}{2}\left(\mathrm{e}^{\frac{\pi}{2}}+1\right)$ Volume $=\pi\left[\int_0^{\frac{\pi}{2}}\left(x+x^2+\frac{1}{3}x^3\right)^2-\int_0^{\frac{\pi}{2}}\left(\mathrm{e}^x\sin x\right)^2\right]\mathrm{d}x$ $\approx 3.19\,units^3$
3	$x^2+3xy+y^3=3$ diff w.r.t x $2x+3x\frac{\mathrm{d}y}{\mathrm{d}x}+3y+3y^2\frac{\mathrm{d}y}{\mathrm{d}x}=0$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2x-3y}{3x+3y^2}$ Tangent // x-axis, $-2x-3y=0$ $y=\frac{-2x}{3}$ Sub $y=\frac{-2x}{3}$ into $x^2+3xy+y^3=3$

$$x^2+3x\left(\frac{-2x}{3}\right)+\left(\frac{-2x}{3}\right)^3=3$$

$$\frac{8}{27}x^3+x^2+3=0$$

$x=-4.00$, $y=2.67$

coordinates $(-4.00, 2.67)$

At $x=-1$, $(-1)^2+3(-1)y+y^3=3$

$$y^3-3y-2=0$$

$$y=2,-1$$

At $(-1,-1)$, $\frac{dy}{dx}$ is undefined, $\therefore$ equation of normal is $y=-1$

At $(-1,2)$, $\frac{dy}{dx}=\frac{-2(-1)-3(2)}{3(-1)+3(2)^2}$

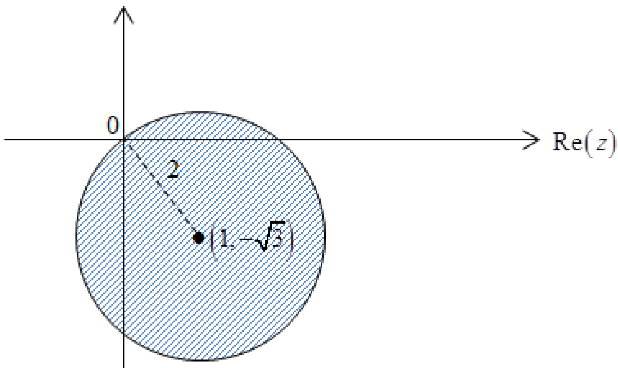
$$\frac{dy}{dx}=\frac{-4}{9}$$

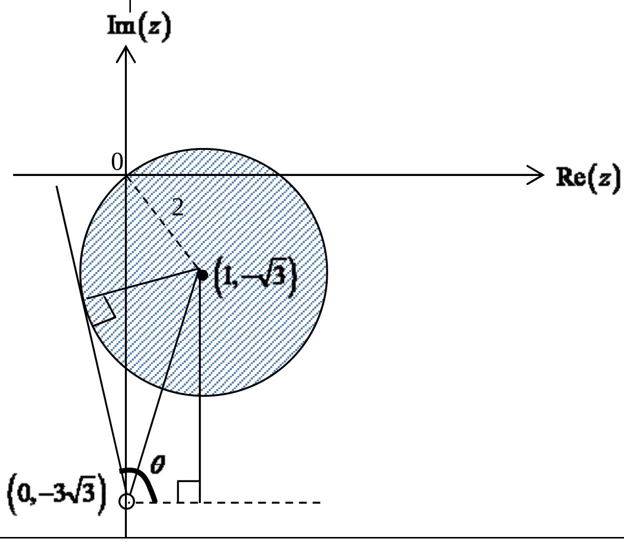
Equation of normal $\frac{y-2}{x+1}=\frac{9}{4}$

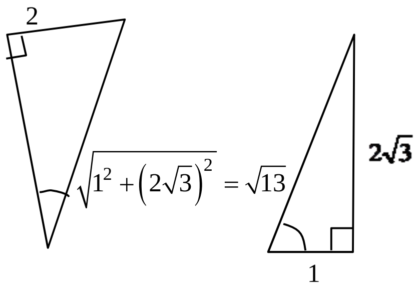
$$y=\frac{9}{4}x+\frac{17}{4}$$

4 **(i)**

ii)





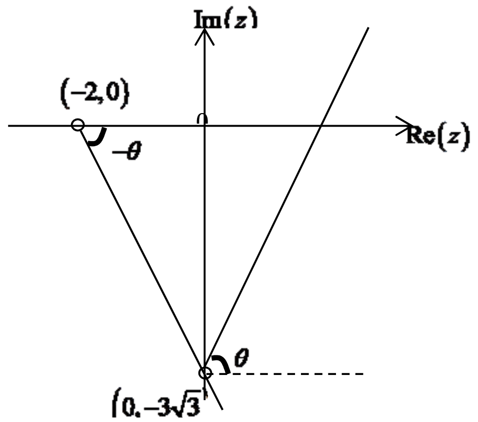


Largest possible value of $\theta = \sin^{-1}\left(\dfrac{2}{\sqrt{13}}\right) + \tan^{-1}\left(\dfrac{2\sqrt{3}}{1}\right)$

$= 1.88 \text{ rad}$ (3 s.f.)

(iii) $\arg\left(w^* + 2\right) = \arg\left(\left(w+2\right)^*\right) = \theta$

$\Rightarrow \quad \arg\left(w+2\right) = -\theta$



Largest value of $\theta = \tan^{-1}\left(\dfrac{3\sqrt{3}}{2}\right)$

$= 1.20 \text{ rad}$

For the half-lines $\arg\left(w + 3\sqrt{3}\text{i}\right) = \theta$ and $\arg\left(w^* + 2\right) = \theta$ to intersect, $0 < \theta < 1.20$ (3 s.f.).

5

Height of an isosceles $\triangle$ = $\sqrt{\left(ax\right)^2 - \left(\dfrac{x}{2}\right)^2} = x\sqrt{a^2 - \dfrac{1}{4}}$

Area of one $\triangle$ $= \dfrac{1}{2}x\left(x\sqrt{a^2-\dfrac{1}{4}}\right) = \dfrac{x^2}{2}\sqrt{a^2-\dfrac{1}{4}}$

Area of base = $6\cdot\dfrac{x^2}{2}\sqrt{a^2-\dfrac{1}{4}} = 3x^2\sqrt{a^2-\dfrac{1}{4}}$

$300 = \left(3x^2\sqrt{a^2-\dfrac{1}{4}}\right)y = \left(3\sqrt{a^2-\dfrac{1}{4}}\right)x^2 y \cdots\cdots (1)$

Surface Area, $S = \left(3x^2\sqrt{a^2-\dfrac{1}{4}}\right)2 + 2xy + 4axy$

From (1), $y = \dfrac{100}{\left(\sqrt{a^2-\dfrac{1}{4}}\right)x^2}$

Therefore

$$S = \left(3x^2\sqrt{a^2-\frac{1}{4}}\right)2 + 2x\frac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x^2} + 4ax\frac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x^2}$$

$$= \left(6\sqrt{a^2-\frac{1}{4}}\right)x^2 + 2\frac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x} + 4a\frac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x}$$

$$= \left(6\sqrt{a^2-\frac{1}{4}}\right)x^2 + \left(\frac{200}{\sqrt{a^2-\frac{1}{4}}} + \frac{400a}{\sqrt{a^2-\frac{1}{4}}}\right)\frac{1}{x}$$

$$\frac{\mathrm{d}S}{\mathrm{d}x} = 2\left(6\sqrt{a^2-\frac{1}{4}}\right)x - \left(\frac{200}{\sqrt{a^2-\frac{1}{4}}} + \frac{400a}{\sqrt{a^2-\frac{1}{4}}}\right)\frac{1}{x^2}$$

Let $\frac{\mathrm{d}S}{\mathrm{d}x} = 0$

$$2\left(6\sqrt{a^2-\frac{1}{4}}\right)x - \left(\frac{200}{\sqrt{a^2-\frac{1}{4}}} + \frac{400a}{\sqrt{a^2-\frac{1}{4}}}\right)\frac{1}{x^2} = 0$$

$$2\left(6\sqrt{a^2-\frac{1}{4}}\right)x = \left(\frac{200}{\sqrt{a^2-\frac{1}{4}}} + \frac{400a}{\sqrt{a^2-\frac{1}{4}}}\right)\frac{1}{x^2}$$

$$x^3 = \frac{200}{12\left(a^2-\frac{1}{4}\right)}\cdot(1+2a)$$

$$x^3 = \frac{200}{12\left(a+\frac{1}{2}\right)\left(a-\frac{1}{2}\right)}\cdot(1+2a)$$

$$x^3 = \frac{200}{3(2a+1)(2a-1)}\cdot(1+2a) = \frac{200}{3(2a-1)}$$

$$x = \left(\frac{200}{3(2a-1)}\right)^{\frac{1}{3}}$$

$$\frac{\mathrm{d}^2S}{\mathrm{d}x^2}=2\left(6\sqrt{a^2-\frac{1}{4}}\right)+2\left(\frac{200}{\sqrt{a^2-\frac{1}{4}}}+\frac{400a}{\sqrt{a^2-\frac{1}{4}}}\right)\frac{1}{x^3}>0$$

as $x>0\Rightarrow x^3>0$ and $a>\frac{1}{2}\Rightarrow a^2-\frac{1}{4}>0$

Recall that $y=\dfrac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x^2}$

therefore $\dfrac{y}{x}=\dfrac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)x^3}=\dfrac{100}{\left(\sqrt{a^2-\frac{1}{4}}\right)\left(\frac{200}{3(2a-1)}\right)}$

$$=\frac{3}{\left(\sqrt{a^2-\frac{1}{4}}\right)\left(\frac{2}{(2a-1)}\right)}=\frac{3(2a-1)}{\sqrt{4a^2-1}}=\frac{3\sqrt{2a-1}}{\sqrt{2a+1}}=3\sqrt{\frac{2a-1}{2a+1}}$$

Method 1(Graphical):

Since $a>\frac{1}{2}$, therefore $0<\dfrac{2a-1}{2a+1}<1$ or $0<\sqrt{\dfrac{2a-1}{2a+1}}<1$, [Draw graph to show],

Therefore $0<3\sqrt{\dfrac{2a-1}{2a+1}}<3$

Method 2(Algebraic)

$$\frac{3\sqrt{2a-1}}{\sqrt{2a+1}}=3\sqrt{\frac{2a-1}{2a+1}}=3\sqrt{1-\frac{2}{2a+1}}$$

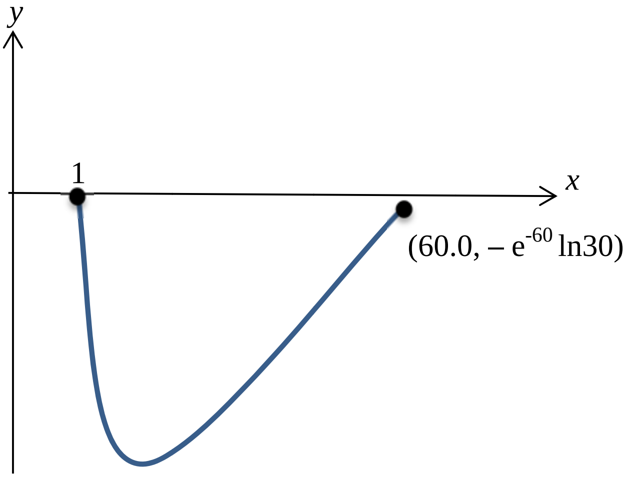
Since $a>\frac{1}{2}$, $\Rightarrow\ 2a+1>2>0$ $\quad\Rightarrow 0<\dfrac{1}{2a+1}<\dfrac{1}{2}$

$$\Rightarrow 0>-\frac{2}{2a+1}>-1$$

$$\Rightarrow 1>1-\frac{2}{2a+1}>0$$

	$$\Rightarrow 0 < \sqrt{1 - \frac{2}{2a+1}} < 1$$ $$\Rightarrow 0 < 3\sqrt{1 - \frac{2}{2a+1}} < 3$$
6	i) Let $w = ye^t$, $\frac{\mathrm{d}w}{\mathrm{d}t} = e^t \frac{\mathrm{d}y}{\mathrm{d}t} + ye^t$ $\frac{\mathrm{d}y}{\mathrm{d}t} + y = \frac{e^{-t}}{t-30}$ $e^t \frac{\mathrm{d}y}{\mathrm{d}t} + ye^t = \frac{1}{t-30}$ $\frac{\mathrm{d}w}{\mathrm{d}t} = \frac{1}{t-30}$ $\int \mathrm{d}w = \int \frac{1}{t-30}\mathrm{d}t = -\int \frac{1}{30-t}\mathrm{d}t$ since $t < 20$ $w = \ln(30-t) + \mathrm{C}$ $ye^t = \ln(30-t) + \mathrm{C}$ $t = 0, y = 0 \Rightarrow 0 = \ln 30 + \mathrm{C}$ So, $\mathrm{C} = -\ln 30$ $y = e^{-t}\left[\ln(30-t) - \ln 30\right]$ $y = e^{-t}\left[\ln\left(\frac{30-t}{30}\right)\right]$ ii) $\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = \mathrm{e}^{-t}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = -\mathrm{e}^{-t} + \mathrm{A}$ $x = \mathrm{e}^{-t} + \mathrm{A}t + \mathrm{B}$ $t = 0$, $\frac{\mathrm{d}x}{\mathrm{d}t} = 2 \Rightarrow 2 = -1 + \mathrm{A}$ So, A = 3 $t = 0, x = 1 \Rightarrow$ B = 0 Hence, $x = \mathrm{e}^{-t}$+3t

iii)



7 **(i)** Let $z = x + y\mathrm{i}$

$$(x + y\mathrm{i})^2 = 4 + 4\sqrt{3}\,\mathrm{i}$$

$$(x^2 - y^2) + 2xy\mathrm{i} = 4 + 4\sqrt{3}\,\mathrm{i}$$

Comparing real and imaginary parts,

$x^2 - y^2 = 4$ ____(1) $2xy = 4\sqrt{3}$ ____(2)

From (2), $x = \dfrac{2\sqrt{3}}{y}$ ____(3)

Sub (3) in (1):

$$\frac{12}{y^2} - y^2 = 4$$

$$y^4 + 4y^2 - 12 = 0$$

$$(y^2 + 2)^2 - 16 = 0$$

$y^2 + 2 = 4$ or $y^2 + 2 = -4$ (rejected since $y \in \mathbb{R}$)

$$y^2 = 2$$

$y = \sqrt{2}$ or $y = -\sqrt{2}$

When $y = \sqrt{2}$, $x = \dfrac{2\sqrt{3}}{\sqrt{2}} = \sqrt{6}$

When $y = -\sqrt{2}$, $x = -\sqrt{6}$

Hence $z = \sqrt{6} + \sqrt{2}\mathrm{i}$ or $z = -\sqrt{6} - \sqrt{2}\mathrm{i}$

Alternative solution:

$$z^2 = 4 + 4\sqrt{3}\,\mathrm{i}$$

$$= 8\mathrm{e}^{\mathrm{i}\frac{\pi}{3}}$$

$$= 8e^{i\left(\frac{\pi}{3}+2n\pi\right)}, \quad n = -1,\ 0$$

$$z = \sqrt{8}e^{i\pi\left(\frac{1+6n}{6}\right)}, \quad n = -1,\ 0$$

$$= \sqrt{8}e^{-i\frac{5\pi}{6}},\ \sqrt{8}e^{i\frac{\pi}{6}}$$

$$= \sqrt{8}\left(\cos\left(-\frac{5\pi}{6}\right)+i\sin\left(-\frac{5\pi}{6}\right)\right),\ \sqrt{8}\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$$

$$= 2\sqrt{2}\left(-\frac{\sqrt{3}}{2}+i\sin\left(-\frac{1}{2}\right)\right),\ 2\sqrt{2}\left(\cos\frac{\sqrt{3}}{2}+i\sin\frac{1}{2}\right)$$

Hence $z = -\sqrt{6}-\sqrt{2}i$ or $z = \sqrt{6}+\sqrt{2}i$

(ii) $(w^*)^3 = 4+4\sqrt{3}i$

$$w^3 = 4-4\sqrt{3}i$$

$$= 8e^{-i\left(\frac{\pi}{3}+2n\pi\right)}$$

$$= 8e^{-i\left(\frac{\pi+6n\pi}{3}\right)}$$

$$w = 2e^{-i\left(\frac{\pi+6n\pi}{9}\right)}, \quad n = -1, 0, 1$$

$$= 2e^{-i\frac{7\pi}{9}}, 2e^{-i\frac{\pi}{9}}, 2e^{i\frac{5\pi}{9}}$$

Method 2:

$$(w^*)^3 = 4+4\sqrt{3}i$$

$$= 8e^{i\frac{\pi}{3}}$$

$$= 8e^{i\left(\frac{\pi}{3}+2n\pi\right)}$$

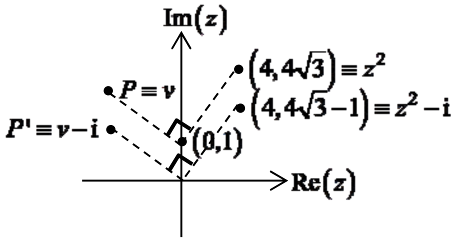
$$= 8e^{i\left(\frac{\pi+6n\pi}{3}\right)}$$

$$w^* = 2e^{i\left(\frac{\pi+6n\pi}{9}\right)}, \quad n = -1, 0, 1$$

$$w = 2e^{-i\frac{7\pi}{9}}, 2e^{-i\frac{\pi}{9}}, 2e^{i\frac{5\pi}{9}}$$

(iii) Since v is obtained by a counter clockwise rotation of the point representing z^2 through one right angle about the point $(0,1)$ on the Argand diagram, $v-i$ is obtained by a counter clockwise rotation of the point representing z^2-i through one right angle about the origin.

Hence
$$v - \mathrm{i} = \mathrm{i}\left(z^2 - \mathrm{i}\right)$$
$$= \mathrm{i}\left(4 + 4\sqrt{3}\,\mathrm{i} - \mathrm{i}\right)$$
$$= \left(1 - 4\sqrt{3}\right) + 4\mathrm{i}$$
$$v = \left(1 - 4\sqrt{3}\right) + 5\mathrm{i}$$



8

(i) Let P_n denotes the proposition: $u_n = \dfrac{\cos nx}{n}$ for all $n \in \mathbb{Z}^+$.

For $n = 1$, LHS = u_1 = $\cos x$ = RHS.

So P_1 is true.

Assume P_k is true for some $k \in \mathbb{Z}^+$. That is, $u_k = \dfrac{\cos kx}{k}$. (IH)

We need to show that assuming that P_k is true, then P_{k+1} must also be true.

That is, we must show that $u_{k+1} = \dfrac{\cos(k+1)x}{k+1}$.

For $n = k + 1$,

$$\text{LHS} = u_{k+1} = u_k - \frac{1}{k(k+1)}\left[2k\sin\frac{x}{2}\sin\left(k+\frac{1}{2}\right)x + \cos kx\right] \text{ by r. r.}$$

$$= \frac{\cos kx}{k} - \frac{1}{k(k+1)}\left[2k\sin\frac{x}{2}\sin\left(k+\frac{1}{2}\right)x + \cos kx\right] \text{ by (IH)}$$

$$= \frac{(k+1)\cos kx - 2k\sin\frac{x}{2}\sin\left(k+\frac{1}{2}\right)x - \cos kx}{k(k+1)}$$

$$= \frac{\cos kx - 2\sin\frac{x}{2}\sin\left(k+\frac{1}{2}\right)x}{k+1}$$

$$= \frac{\cos kx + \left[\cos(k+1)x - \cos kx\right]}{k+1} \quad \text{(By Factor Formula)}$$

$$= \frac{\cos(k+1)x}{k+1} = \text{RHS}$$

Thus P_{k+1} is true.

Since P_1 is true and P_k is true $\Rightarrow$ P_{k+1} is true, by MI, P_n is true.

(ii) $\displaystyle\sum_{n=1}^{4N}\left(\frac{1}{n}\cos\frac{n\pi}{2}\right)$

$$= \underbrace{\cos\frac{\pi}{2}}_{0} + \frac{1}{2}\cos\pi + \underbrace{\frac{1}{3}\cos\frac{3\pi}{2}}_{0} + \frac{1}{4}\cos 2\pi + \ldots + \frac{1}{4N}\cos 2N\pi$$

$$= -\frac{1}{2} + \frac{1}{4} - \frac{1}{6} + \frac{1}{8} - \ldots + \frac{1}{4N}$$

$$= \sum_{n=1}^{2N} \frac{(-1)^n}{2n} = \frac{1}{2}\sum_{n=1}^{2N} \frac{(-1)^n}{n}.$$

(iii) By **(ii)**, $\displaystyle\sum_{n=1}^{\infty} \left(\frac{1}{n}\cos\frac{n\pi}{2}\right) = \frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$

$$= \frac{1}{2}\left(-1+\frac{1}{2}-\frac{1}{3}+\ldots\right)$$

$$= -\frac{1}{2}\left(1-\frac{1}{2}+\frac{1}{3}-\ldots\right)$$

$$= -\frac{1}{2}\ln 2$$

by putting $x = 1$ in the Maclaurin's expansion of $\ln(1+x)$ in MF15 which gives $1-\frac{1}{2}+\frac{1}{3}-\ldots = \ln 2$.

9 A direction vector parallel to l_3 is given by $\alpha\mathbf{b}-\beta\mathbf{a}$

Since l_3 is perpendicular to l_1, $\mathbf{a}\cdot(\alpha\mathbf{b}-\beta\mathbf{a}) = 0 \Rightarrow \beta = \dfrac{\alpha\mathbf{a}.\mathbf{b}}{|\mathbf{a}|^2}$

Therefore, l_3 is parallel to $\alpha\mathbf{b}-\dfrac{\alpha\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$, i.e. $\mathbf{b}-\left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a}$

Direction vector of l_3 is given by $\mathbf{b}-\left(\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\right)\mathbf{a} = \begin{pmatrix}17\\3\\4\end{pmatrix} - \dfrac{\begin{pmatrix}11\\10\\2\end{pmatrix}.\begin{pmatrix}17\\3\\4\end{pmatrix}}{\left|\begin{pmatrix}11\\10\\2\end{pmatrix}\right|^2}\begin{pmatrix}11\\10\\2\end{pmatrix} = \begin{pmatrix}6\\-7\\2\end{pmatrix}$

Equation of l_3 is $\mathbf{r} = \begin{pmatrix}3\\-5\\2\end{pmatrix}+\lambda\begin{pmatrix}6\\-7\\2\end{pmatrix}$, where $\lambda \in \mathbb{R}$.

Clearly, the direction vectors $\begin{pmatrix}6\\-7\\2\end{pmatrix}$ and $\begin{pmatrix}-3\\4\\-1\end{pmatrix}$ are non-parallel. Hence the two lines are not parallel.

Equating $\begin{pmatrix}3+6\lambda\\-5-7\lambda\\2+2\lambda\end{pmatrix} = \begin{pmatrix}10-3\mu\\-3+4\mu\\1-\mu\end{pmatrix}$,

there are no unique values for λ and μ that satisfy the 3 equations. Therefore l_3 and l_4 are skew.

	$$\begin{pmatrix}6\\-7\\2\end{pmatrix}\times\begin{pmatrix}-3\\4\\-1\end{pmatrix}=\begin{pmatrix}-1\\0\\3\end{pmatrix}$$ Vector normal to both l_3 and l_4 is $\begin{pmatrix}-1\\0\\3\end{pmatrix}$, which is direction vector of l_5 $$\begin{pmatrix}11\\10\\2\end{pmatrix}\times\begin{pmatrix}17\\3\\4\end{pmatrix}=\begin{pmatrix}34\\-10\\-137\end{pmatrix}$$ Vector normal to plane containing l_1 and l_2 is $\begin{pmatrix}-34\\10\\137\end{pmatrix}$ Let required angle be θ. Using $\sin\theta=\dfrac{\begin{pmatrix}-1\\0\\3\end{pmatrix}.\begin{pmatrix}-34\\10\\137\end{pmatrix}}{\sqrt{10\times 20025}}\Rightarrow\theta=83.9^\circ$
10	$x=1+\cos\theta\Rightarrow\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-\sin\theta$ $y=2\sin\theta\Rightarrow\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=2\cos\theta$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=-2\cot\theta$ Equation of tangent: $y-2\sin\theta=-2\cot\theta\left(x-1-\cos\theta\right)$ $y=-2\cot\theta\left(x-1-\cos\theta\right)+2\sin\theta$ $=2\left[\dfrac{-\cos\theta\left(x-1-\cos\theta\right)+\sin^2\theta}{\sin\theta}\right]$ $=2\left[\dfrac{-\cos\theta x+\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta}\right]$ $=2\left(-\cot\theta x+\cot\theta+\operatorname{cosec}\theta\right)$ At P, $\theta=\dfrac{5\pi}{6}$. Equation of tangent at P: $y=2\left(\sqrt{3}x+2-\sqrt{3}\right)$ At Q, $\theta=\dfrac{\pi}{6}$. Equation of tangent at Q: $y=2\left(-\sqrt{3}x+2+\sqrt{3}\right)$ The 2 tangents meet at R: $2\left(\sqrt{3}x+2-\sqrt{3}\right)=2\left(-\sqrt{3}x+2+\sqrt{3}\right)$ $x=1$ and $y=4$

Hence the y-coordinate of point R is 4.

Equation of tangent at P: $y = 2\left(\sqrt{3}x + 2 - \sqrt{3}\right)$

At $y = 0$, $x = \dfrac{\sqrt{3}-2}{\sqrt{3}}$

Equation of tangent at Q: $y = 2\left(-\sqrt{3}x + 2 + \sqrt{3}\right)$

At $y = 0$, $x = \dfrac{\sqrt{3}+2}{\sqrt{3}}$

$$A = \frac{1}{2} \times 4 \times \left(\frac{\sqrt{3}+2}{\sqrt{3}} - \frac{\sqrt{3}-2}{\sqrt{3}} \right) = \frac{8}{\sqrt{3}}$$

$$B = \int_0^2 y \;\; \mathrm{d}x$$

$$= 2\int_\pi^0 \sin\theta \left(-\sin\theta \mathrm{d}\theta\right)$$

$$= 2\int_0^\pi \sin^2\theta \,\mathrm{d}\theta$$

$$= \int_0^\pi \left(1 - \cos 2\theta\right) \mathrm{d}\theta$$

$$= \left[\theta - \frac{\sin 2\theta}{2}\right]_0^\pi = \pi$$

$$A - B = \frac{8}{\sqrt{3}} - \pi$$

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