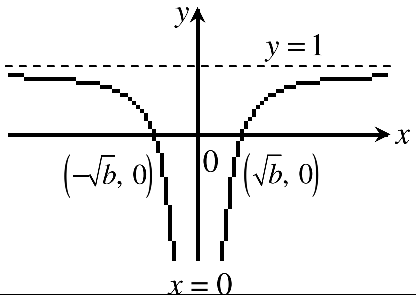
<table>
<tr><th>Qn.
[Marks]</th><th>Solution</th></tr>
<tr><td>1

[4]</td><td>

$\mathrm{f}(x) = \dfrac{x^2 - ax + b}{x} = x - a + \dfrac{b}{x}$

$\mathrm{f}'(x) = 1 - \dfrac{b}{x^2} = 0 \Leftrightarrow x = \pm\sqrt{b}$

x-intercepts $\left(\sqrt{b}, 0\right)$ *and* $\left(-\sqrt{b}, 0\right)$

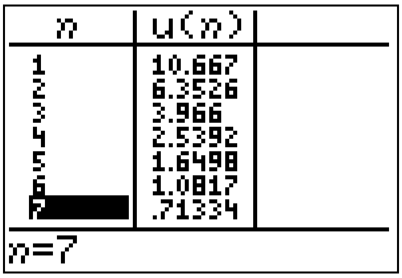


</td></tr>
<tr><td>2(i)

[2]</td><td>

Using G.C.

<table>
<tr><td>$n = 6$</td><td>$u_6 = 1.0817 > 1$</td></tr>
<tr><td>$n = 7$</td><td>$u_7 = 0.71334 < 1$</td></tr>
</table>



Least $n = 7$

</td></tr>
<tr><td>(ii)

[2]</td><td>

Since $u_n = 20$ for all values of n, then

$20 = r(20)\left(1 - \dfrac{20}{100}\right) \Rightarrow r = \dfrac{1}{\left(1 - \dfrac{20}{100}\right)} = \dfrac{5}{4}$

</td></tr>
<tr><td>(iii)

[2]</td><td>

As $n \to \infty,\ u_n \to l$ and $u_{n+1} \to l,\ \ l = \dfrac{4}{3}l\left(1 - \dfrac{l}{100}\right)$

$l = 0$ (N.A.) or $\dfrac{4}{3}\left(1 - \dfrac{l}{100}\right) = 1$

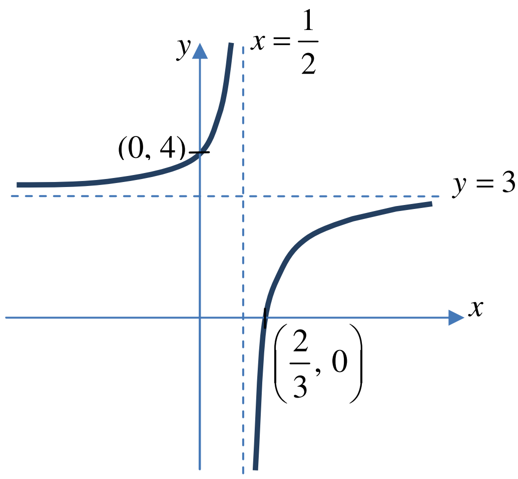
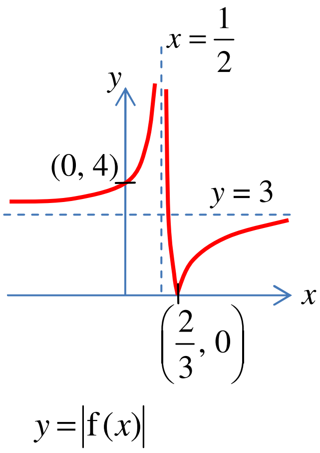
$l = 25$

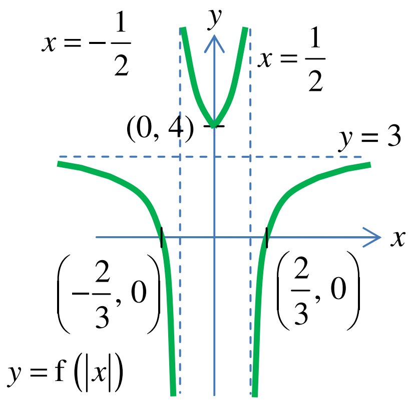
</td></tr>
</table>

Qn. [Marks]	Solution
3(i) **[3]**	$\mathrm{f}(x)=(1-x)\left(1+x^2\right)^{-2}$ $=(1-x)\left[1+\frac{(-2)}{1!}x^2+\frac{(-2)(-3)}{2!}x^4+\cdots\right]$ $=(1-x)\left(1-2x^2+3x^4+\cdots\right)$ $=1-x-2x^2+2x^3+3x^4+\cdots$
(ii) **[3]**	Observe that $\left(1+x^2\right)^{-2}=1+\frac{(-2)}{1!}x^2+\frac{(-2)(-3)}{2!}x^4+\cdots+\frac{(-2)(-3)\cdots(-r-1)}{r!}x^{2r}+\cdots$ $=1-2x^2+3x^4+\cdots+(-1)^r(r+1)x^{2r}+\cdots$ Since $\mathrm{f}(x)=(1-x)\left(1+x^2\right)^{-2}$, the coefficient of x^{2r+1} is $-(-1)^r(r+1)=(-1)^{r+1}(r+1)$.
4 **[4]**	Let P_n be the statement $\sum_{r=1}^{n}\sin(2rx)\sin x=\sin nx\sin(n+1)x$ for $n\in\mathbb{Z}^+$. When n = 1, L.H.S. $=\sum_{r=1}^{1}\sin(2rx)\sin x=\sin 2x\sin x$ R.H.S. = $\sin x\sin 2x$ ∴ L.H.S. = R.H.S. Hence P_1 is true. Assume P_k is true for some $k\in\mathbb{Z}^+$, i.e. $\sum_{r=1}^{k}\sin(2rx)\sin x=\sin kx\sin(k+1)x$ To prove that P_{k+1} is true, i.e. $\sum_{r=1}^{k+1}\sin(2rx)\sin x=\sin(k+1)x\sin(k+2)x$ L.H.S. $=\sum_{r=1}^{k+1}\sin(2rx)\sin x$ $=\sum_{r=1}^{k}\sin(2rx)\sin x+\sin 2(k+1)x\sin x$ $=\sin kx\sin(k+1)x+\sin 2(k+1)x\sin x$ $=\sin kx\sin(k+1)x+\left[2\sin(k+1)x\cos(k+1)x\right]\sin x$

Qn. [Marks]	Solution
	$= \sin(k+1)x\left[\sin kx + 2\cos(k+1)x\sin x\right]$ $= \sin(k+1)x\left[\sin kx + \sin(k+2)x - \sin kx\right]$ $= \sin\left(k+2\right)x\sin\left(k+1\right)x$ R.H.S. **OR** L.H.S. $= \sum_{r=1}^{k+1}\sin(2rx)\sin x$ $= \sum_{r=1}^{k}\sin(2rx)\sin x + \sin 2(k+1)x\sin x$ $= \sin kx\sin(k+1)x + \sin 2(k+1)x\sin x$ $= -\frac{1}{2}\left[\cos(2k+1)x - \cos x\right] - \frac{1}{2}\left[\cos(2k+3)x - \cos(2k+1)x\right]$ $= -\frac{1}{2}\left[\cos(2k+3)x - \cos x\right]$ $= -\frac{1}{2}\left[-2\sin\left(\frac{2k+3+1}{2}\right)x\sin\left(\frac{2k+3-1}{2}\right)x\right]$ $= \sin\left(k+2\right)x\sin\left(k+1\right)x$ = R.H.S. Hence P_k is true $\Rightarrow$ P_{k+1} is true, and since P_1 is true, by mathematical induction , P_n is true for all $n \in \mathbb{Z}^+$.
[3]	$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \frac{\sin 3x \sin 4x}{\sin x}\,\mathrm{d}x$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sum_{r=1}^{3} \sin(2rx)\ \mathrm{d}x.$ $= \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \left(\sin 2x + \sin 4x + \sin 6x\right)\ \mathrm{d}x.$ $= \left[-\frac{1}{2}\cos 2x - \frac{1}{4}\cos 4x - \frac{1}{6}\cos 6x\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$ $= -\frac{1}{2}(-1-0) - \frac{1}{4}(1+1) - \frac{1}{6}(-1-0) = \frac{1}{6}$

<table>
<tr><th>Qn.
[Marks]</th><th>Solution</th></tr>
<tr><td>5

[4]</td><td>Equation of the new curve is $y=3-\dfrac{1}{2x-1}$.

</td></tr>
<tr><td>[4]</td><td>



From the graph, for $\lvert \mathrm{f}(x)\rvert > \mathrm{f}\left(\lvert x\rvert\right)$, $x<-\dfrac{1}{2}$ or $\dfrac{1}{2}<x<\dfrac{2}{3}$.</td></tr>
<tr><td>6(i)

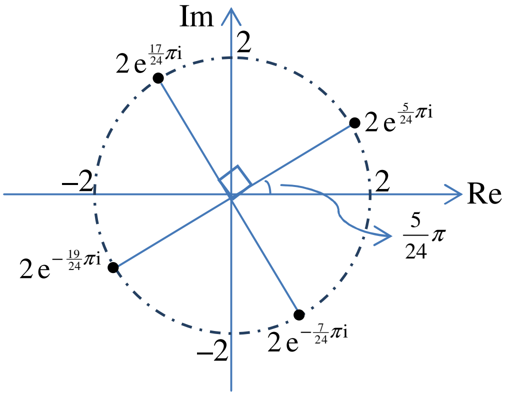
[1]</td><td>$\left\lvert \mathbf{a}\bullet\dfrac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right\rvert$ represents the length of projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$.</td></tr>
<tr><td>(ii)

[2]</td><td>$$\lvert 3\mathbf{a}-\mathbf{b}\rvert^2=10^2=100$$ $$9\lvert\mathbf{a}\rvert^2+\lvert\mathbf{b}\rvert^2-6\mathbf{a}\bullet\mathbf{b}=100$$ $$6\mathbf{a}\bullet\mathbf{b}=9(3)^2+(5)^2-100=6$$ Therefore $\mathbf{a}\bullet\mathbf{b}=1$.</td></tr>
<tr><td>(iii)

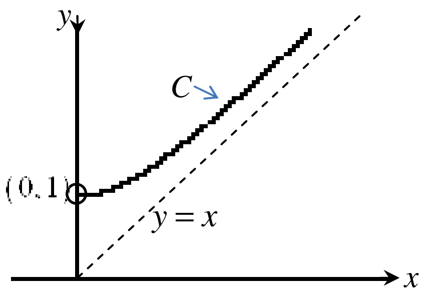
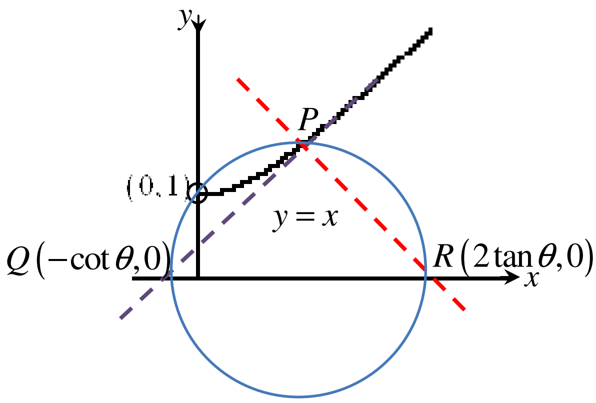
[2]</td><td>Let N be the foot of the perpendicular from A to the line OB.
$$ON=\left\lvert \mathbf{a}\bullet\dfrac{\mathbf{b}}{\lvert\mathbf{b}\rvert}\right\rvert=\frac{1}{5}.$$ Using Pythagoras Theorem,</td></tr>
</table>

Qn. [Marks]	Solution
	$AN^2 = OA^2 - ON^2 = 3^2 - \left(\frac{1}{5}\right)^2 = \frac{224}{25}.$ $AN = \sqrt{\frac{224}{25}} = \frac{4}{5}\sqrt{14} = 2.9933 = 2.99$ (3sf). Area of triangle $OAB = \frac{1}{2}OB \times AN = 2\sqrt{14} = 7.48$ (3sf).
(iv) **[4]**	$(\mu\mathbf{a}+2\mathbf{b})-\mathbf{a} = k\left[(2\mathbf{a}+3\mathbf{b})-\mathbf{a}\right]$ for some constant k. Now, $\lvert\mathbf{a}\rvert \neq 0$, $\lvert\mathbf{b}\rvert \neq 0$ and $\lvert\mathbf{a}\cdot\mathbf{b}\rvert = 1 \neq \lvert\mathbf{a}\rvert\lvert\mathbf{b}\rvert$, so **a** and **b** are non-zero and non-parallel vectors. $(\mu-1)\mathbf{a}+2\mathbf{b} = k\mathbf{a}+3k\mathbf{b}$ $(\mu-1-k)\mathbf{a} = (3k-2)\mathbf{b}$ Hence $3k-2=0 \Rightarrow k=\frac{2}{3}$ and $\mu = k+1 = \frac{5}{3}$.
7(a) **[2]**	C_1 is a circle centred at (0,0) with radius 5. $C_2 : \frac{x^2}{100/a} + \frac{y^2}{100/b} = 1$ is an ellipse centred at (0,0) with length of the horizontal axis $2(\frac{10}{\sqrt{a}})$ and vertical axis $2(\frac{10}{\sqrt{b}})$. **Note :** $a < b \Rightarrow$ length of the horizontal axis > length of vertical axis To get 4 points of intersection, we need : $\frac{10}{\sqrt{a}} > 5 \Rightarrow 0 < a < 4$ and $\frac{10}{\sqrt{b}} < 5 \Rightarrow b > 4$ **OR** Compare $C_1 : x^2 + y^2 = 25 \Rightarrow \frac{x^2}{25} + \frac{y^2}{25} = 1$ with $C_2 : \frac{x^2}{100/a} + \frac{y^2}{100/b} = 1$. For them to intersects at 4 points, $\frac{100}{b} < 25$ and $\frac{100}{a} > 25$ $b > 4$ and $0 < a < 4$ since $a > 0$ is given.

Qn. [Marks]	Solution
(b) **[3]**	$C_1: \quad x^2+y^2=25 \quad \Rightarrow \quad y=\pm\sqrt{25-x^2}$ $C_2: \dfrac{x^2}{10^2}+\dfrac{y^2}{\left(10/3\right)^2}=1 \quad \Rightarrow \quad y=\pm\dfrac{\sqrt{100-x^2}}{3}$ C_1 and C_2 intersect at $x=\pm3.9528$ (5 s.f.) (from GC) Thus area of the required region $=2\left[\int_{-10}^{-3.9528}\dfrac{\sqrt{100-x^2}}{3}\ \mathrm{d}x-\int_{-5}^{-3.9528}\sqrt{25-x^2}\ \mathrm{d}x\right]$ $=22.3$ (3 s.f.) **OR** $C_1: \quad x^2+y^2=25 \quad \Rightarrow \quad x=\pm\sqrt{25-y^2}$ $C_2: x^2+9y^2=100 \quad \Rightarrow \quad x=\pm\sqrt{100-9y^2}$ C_1 and C_2 intersect at $y=\pm3.0619$ (5 s.f.) (from GC) Thus area of the required region $=2\left[\int_{0}^{3.0619}\sqrt{100-9y^2}-\sqrt{25-y^2}\ \mathrm{d}y\right]$ $=22.3$ (3 s.f.)
(c) **[4]**	$C_1: \quad x^2+y^2=25 \quad \Rightarrow \quad x^2=25-y^2$ $C_2: \dfrac{x^2}{10^2}+\dfrac{y^2}{\left(10/2\right)^2}=1 \quad \Rightarrow \quad x^2=100-4y^2$ Required Volume $=\pi\int_{-5}^{5}x^2\ \mathrm{d}y-\dfrac{4}{3}\pi(5)^3$ **Note:** $\dfrac{4}{3}\pi(5)^3$ is the volume of sphere $=\pi\int_{-5}^{5}\left(100-4y^2\right)\ \mathrm{d}y-\dfrac{500}{3}\pi$ formed when rotating the circle $=\pi\left[100y-\dfrac{4}{3}y^3\right]_{-5}^{5}-\dfrac{500}{3}\pi$ about the $y-$axis. $=\pi\left[\left(500-\dfrac{500}{3}\right)-\left(-500+\dfrac{500}{3}\right)\right]-\dfrac{500}{3}\pi$ $=500\pi$

Qn. [Marks]	Solution
8(a) **[6]**	$z^4-8\left(-\sqrt{3}+\mathrm{i}\right)=0$ $z^4=8\left(-\sqrt{3}+\mathrm{i}\right)$ $=16\mathrm{e}^{\frac{5}{6}\pi\mathrm{i}}$ $=16\mathrm{e}^{\left(\frac{5}{6}+2k\right)\pi\mathrm{i}}$ $z=2\mathrm{e}^{\left(\frac{5}{24}+\frac{1}{2}k\right)\pi\mathrm{i}}$, $k=0$, ±1, -2 
(b) **[6]**	$w^2+aw^*+b=0$ $\left(w^2+aw^*+b\right)^*=0^*$ $\left(w^2\right)^*+\left(aw^*\right)^*+b^*=0$ $\left(w^*\right)^2+a\left(w^*\right)^*+b=0$, $a^*=a$ and $b^*=b$ since a and b are real. Hence, w^* is a root of $z^2+az^*+b=0$. $z^2+6z^*+9=0$ $\left(x+\mathrm{i}y\right)^2+6\left(x-\mathrm{i}y\right)+9=0$ $x^2-y^2+2\mathrm{i}xy+6x-6\mathrm{i}y+9=0$ $x^2-y^2+6x+9+2y\left(x-3\right)\mathrm{i}=0$ Compare imaginary parts, $y=0$ or $x=3$. Consider real parts: When $y=0$, $x^2+6x+9=0$ which gives $x=-3$ When $x=3$, $3^2-y^2+18+9=0$ giving $y=\pm6$ Hence $z=-3$, $3+6\mathrm{i}$, $3-6\mathrm{i}$

Qn. [Marks]	Solution
9 (i) **[3]**	$\overrightarrow{OA}=\begin{pmatrix}5\\0\\0\end{pmatrix}$, $\overrightarrow{OC}=\begin{pmatrix}0\\3\\0\end{pmatrix}$, $\overrightarrow{OR}=\begin{pmatrix}5\\3\\2\end{pmatrix}$, so $\mathbf{n}=\overrightarrow{AR}\times\overrightarrow{CR}=\begin{pmatrix}0\\3\\2\end{pmatrix}\times\begin{pmatrix}5\\0\\2\end{pmatrix}=\begin{pmatrix}6\\10\\-15\end{pmatrix}$. Therefore π_1 : $\mathbf{r}\bullet\begin{pmatrix}6\\10\\-15\end{pmatrix}=\begin{pmatrix}5\\0\\0\end{pmatrix}\bullet\begin{pmatrix}6\\10\\-15\end{pmatrix}=30.$
(ii) **[2]**	The angle between π_1 and the horizontal base, which has normal $\begin{pmatrix}0\\0\\1\end{pmatrix}$, is $\cos^{-1}\dfrac{\begin{pmatrix}6\\10\\-15\end{pmatrix}.\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{36+100+225}\sqrt{0+0+1}}=\cos^{-1}\left(\dfrac{-15}{19}\right)=142.136^\circ$. The acute angle is $180^\circ-142.136^\circ=37.9^\circ$. (1 dec. pl.)
(iii) **[1]**	By symmetry, the acute angle between π_2 and the horizontal base is also 37.864° Hence the angle between π_1 and π_2 is $=2(37.864^\circ)$ = 75.7° (1 dec. pl.).
(iv) **[3]**	$\overrightarrow{OP}=\begin{pmatrix}0\\0\\2\end{pmatrix}$, $\overrightarrow{OS}=\begin{pmatrix}0\\3\\2\end{pmatrix}$, $\overrightarrow{SP}=\begin{pmatrix}0\\-3\\0\end{pmatrix}$, therefore $\overrightarrow{OX}=\overrightarrow{OS}+\overrightarrow{SX}=\begin{pmatrix}0\\3\\2\end{pmatrix}+\alpha\begin{pmatrix}0\\-3\\0\end{pmatrix}=\begin{pmatrix}0\\3(1-\alpha)\\2\end{pmatrix}$. l_{XY} : $\mathbf{r}=\overrightarrow{OY}+\lambda\overrightarrow{XY},\ \lambda\in\mathbb{R}$, and with $\overrightarrow{OY}=\begin{pmatrix}5\\2\\1\end{pmatrix}$, l_{XY} : $\mathbf{r}=\begin{pmatrix}5\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}5\\3\alpha-1\\-1\end{pmatrix},\ \lambda\in\mathbb{R}.$

Qn. [Marks]	Solution
	Clearly l_{OR} : $\mathbf{r}=\mu\begin{pmatrix}5\\3\\2\end{pmatrix}$, $\mu\in\mathbb{R}$.
(v) **[4]**	When the lines XY and OR intersect, $\mu\begin{pmatrix}5\\3\\2\end{pmatrix}=\begin{pmatrix}5\\2\\1\end{pmatrix}+\lambda\begin{pmatrix}5\\3\alpha-1\\-1\end{pmatrix}$. Therefore, $\mu=1+\lambda$ $3\mu=2+\lambda(3\alpha-1)$ $2\mu=1-\lambda$ Solving gives $\mu=\frac{2}{3}$, $\lambda=-\frac{1}{3}$ and $\alpha=\frac{1}{3}$. $\overrightarrow{OW}=\mu\overrightarrow{OR}$, so $OW:OR=2:3$.
10(i) **[3]**	Since $x=\tan\theta$, $y=\sec\theta$ and $\tan^2\theta+1=\sec^2\theta$ $x^2+1=y^2$ for $x>0$ and $y>1$ 
(ii) **[7]**	 $x=\tan\theta,\ y=\sec\theta$; $\frac{\mathrm{dy}}{\mathrm{dx}}=\frac{\sec\theta\tan\theta}{\sec^2\theta}=\frac{\tan\theta}{\sec\theta}=\sin\theta$ Equation of tangent at P is $y-\sec\theta=\sin\theta\left(x-\tan\theta\right)$

Qn. [Marks]	Solution
	At Q, $0-\sec\theta=\sin\theta\left(x-\tan\theta\right)$. $x=\dfrac{\sin\theta}{\cos\theta}-\dfrac{1}{\sin\theta\cos\theta}=\dfrac{\sin^2\theta-1}{\sin\theta\cos\theta}=-\dfrac{\cos^2\theta}{\sin\theta\cos\theta}$ $x=-\cot\theta$ Tangent at P intersects *x*-axis at $Q\left(-\cot\theta,0\right)$ Equation of normal at *P* is $y-\sec\theta=\dfrac{-1}{\sin\theta}\left(x-\tan\theta\right)$ At R, $0-\sec\theta=\dfrac{-1}{\sin\theta}\left(x-\tan\theta\right)$ $x=\dfrac{\sin\theta}{\cos\theta}+\tan\theta=2\tan\theta$ Normal at *P* intersects *x*-axis at $R\left(2\tan\theta,0\right)$ Triangle PQR is a right-angled triangle in circle, so QR is a diameter. $A=\pi\left(\dfrac{QR}{2}\right)^2=\pi\left(\dfrac{2\tan\theta-\left(-\cot\theta\right)}{2}\right)^2$ $=\pi\left(\tan\theta+\dfrac{1/\tan\theta}{2}\right)^2$ $A=\pi\left(\tan\theta+\dfrac{1}{2\tan\theta}\right)^2$ [shown]
(iii) **[2]**	$\left(t+\dfrac{1}{2t}\right)^2-2=t^2+1+\dfrac{1}{4t^2}-2=t^2-1+\dfrac{1}{4t^2}$ $=\left(t-\dfrac{1}{2t}\right)^2\geq 0$
[1]	For $0<\theta<\dfrac{\pi}{2}$, $t=\tan\theta>0$ So minimizing $A=\pi\left(\tan\theta+\dfrac{1}{2\tan\theta}\right)^2$ over $0<\theta<\dfrac{\pi}{2}$ is equivalent to minimizing $A=\pi\left(t+\dfrac{1}{2t}\right)^2$ for $t>0$ $A=\pi\left(\tan\theta+\dfrac{1}{2\tan\theta}\right)^2\geq 2\pi$ from above result. Hence the minimum value of $A=2\pi$

Qn. [Marks]	Solution
11(i) **[1]**	When $x = 0$, $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -1$. (since $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -y$ and $y = 1$ when $x = 0$)
(ii) **[2]**	Differentiating (1) with respect to x twice gives $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4} = -\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2}$. When $x=0$, $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 1$ (given) and $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -1$ [from (**i**)] Hence, when $x=0$, $\dfrac{\mathrm{d}^3 y}{\mathrm{d}x^3} = -1$ and $\dfrac{\mathrm{d}^4 y}{\mathrm{d}x^4} = 1$. Maclaurin's theorem gives $y = 1 + \dfrac{1}{1!}x^1 + \dfrac{-1}{2!}x^2 + \dfrac{-1}{3!}x^2 + \dfrac{1}{4!}x^4 + \cdots$ $y = 1 + x - \dfrac{1}{2}x^2 - \dfrac{1}{6}x^3 + \dfrac{1}{24}x^4 + \cdots$
(iii) **[4]**	Differentiating $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{u}$ implicitly with respect to x gives $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{1}{2}\dfrac{1}{\sqrt{u}}\dfrac{\mathrm{d}u}{\mathrm{d}x}$ $-y = \dfrac{1}{2}\dfrac{\mathrm{d}x}{\mathrm{d}y}\dfrac{\mathrm{d}u}{\mathrm{d}x}$ (since $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -y$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{u} \Rightarrow \dfrac{1}{\sqrt{u}} = \dfrac{\mathrm{d}x}{\mathrm{d}y}$) Hence, $\dfrac{\mathrm{d}u}{\mathrm{d}y} = -2y$. (shown)
	Alternative Method $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{u} \Rightarrow u = \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$. Differentiating $u = \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2$ implicitly with respect to x gives $\dfrac{\mathrm{d}u}{\mathrm{d}x} = 2\dfrac{\mathrm{d}y}{\mathrm{d}x}\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}x}\dfrac{\mathrm{d}x}{\mathrm{d}y} = 2(-y) \Rightarrow \dfrac{\mathrm{d}u}{\mathrm{d}y} = -2y$. (shown)
	Integrating $\dfrac{\mathrm{d}u}{\mathrm{d}y} = -2y$ with respect to y gives $u = \int -2y\,\mathrm{d}y = -y^2 + A$, where A is an arbitrary constant.

Qn. [Marks]	Solution
	Using $u=1$ when $y=1$, we have $A=2$, Hence, $u=2-y^2$.
(iv) **[4]**	Substituting $u=2-y^2$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{u}$ gives $$\frac{\mathrm{d}y}{\mathrm{d}x}=\sqrt{2-y^2},$$ which we can integrate via $$\frac{1}{\sqrt{2-y^2}}\frac{\mathrm{d}y}{\mathrm{d}x}=1$$ $$\int\frac{1}{\sqrt{2-y^2}}\,\mathrm{d}y=\int 1\,\mathrm{d}x$$ $$\sin^{-1}\frac{y}{\sqrt{2}}=x+B$$ $$y=\sqrt{2}\sin\left(x+B\right), \text{ where } B \text{ is an arbitrary constant}$$ Using $y=1$ when $x=0$, we have $B=\sin^{-1}\frac{1}{\sqrt{2}}=\frac{\pi}{4}$. Hence $y=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$, i.e. $P=\sqrt{2}$ and $Q=\frac{\pi}{4}$.
(v) **[2]**	$$\begin{aligned} y&=\sqrt{2}\sin\left(x+\frac{\pi}{4}\right)\\ &=\sqrt{2}\left[\sin x\cos\frac{\pi}{4}+\cos x\sin\frac{\pi}{4}\right]\\ &=\sin x+\cos x \quad (\text{since } \cos\frac{\pi}{4}=\sin\frac{\pi}{4}=\frac{1}{\sqrt{2}})\\ &=\left(x-\frac{x^3}{3!}+\cdots\right)+\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots\right) \quad (\text{from MF15})\\ &=1+x-\frac{x^2}{2}-\frac{x^3}{6}+\frac{x^4}{24}+\cdots \end{aligned}$$