NANYANG JUNIOR COLLEGE
JC2 PRELIMINARY EXAMINATION

Higher 2

MATHEMATICS **9740/02**

Paper 2 **17th September 2014**

3 Hours

Additional Materials: Cover Sheet
Answer Paper
Graph Paper
List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

Section A: Pure Mathematics [40 marks]

1 The three vectors **l**, **m** and **n** are such that $\mathbf{n} = \mathbf{l} \times \mathbf{m}$**.** Show that $\mathbf{l} \cdot \mathbf{n} = 0$. **[1]**

With respect to an origin *O*, the position vectors of three non-collinear points *A*, *B* and *C* are **a**, **b** and **c** respectively. The scalars λ and μ are such that $\lambda\mathbf{a} + \mu\mathbf{b}$ is the projection of **c** onto the plane containing *O*, *A* and *B*.

By considering $\overrightarrow{CC'}$, where *C'* is the foot of perpendicular from *C* to the plane containing *O*, *A* and *B*, explain why there is a scalar *t* such that $t(\mathbf{a} \times \mathbf{b}) = \mathbf{c} - \lambda\mathbf{a} - \mu\mathbf{b}$, and deduce that $\lambda(\mathbf{a} \cdot \mathbf{a}) + \mu(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot \mathbf{c}$. **[4]**

2 The functions f, g and h are defined by

$$\mathrm{f} : x \mapsto (x-a)(x-2), \qquad x \in \mathbb{R}, \text{ where } a < -2,$$
$$\mathrm{g} : x \mapsto \mathrm{e}^{x}, \qquad x \in \mathbb{R},$$
$$\mathrm{h} : x \mapsto \ln(x-2), \qquad x \in \mathbb{R}, x > 3.$$

(i) Show that the composite function gh exists and define gh in a similar form. State the range of gh. **[4]**

(ii) Find, in terms of *a*, the exact range of *x* for which $\mathrm{f}(x+a) > \mathrm{gh}(x+a)$. **[3]**

(iii) The curve with equation $y = \mathrm{f}(x)$ has a stationary point with coordinates $\left(\frac{2+a}{2}, -\frac{(a-2)^2}{4}\right)$.

Sketch the graph of $y = \frac{1}{\mathrm{f}(x)}$, stating clearly the equations of any asymptotes and the coordinates of the stationary point. **[3]**

3 Given that $y = \frac{\ln\sqrt{1+x}}{1+x}$, where $-1 < x < 1$, show that $2(1+x)\frac{\mathrm{d}y}{\mathrm{d}x} + 2y = \frac{1}{1+x}$. **[1]**

(i) By further differentiation, find the Maclaurin series for *y* up to and including the term in x^3. **[5]**

(ii) Verify that the same result is obtained if the standard series expansions are used. **[3]**

(iii) Deduce the approximate value of $\int_0^{\frac{1}{4}\pi} y \, \mathrm{d}x$. Explain why the approximation is not good. **[2]**

(iv) State the equation of the tangent to the curve *y* at $x = 0$. **[1]**

4 **(a)** The sequence of triangular numbers $\{t_n\}$ is given by the following recurrence relation:

$$t_{n+1} - t_n = n + 1,\ n \in \mathbb{Z}^+ \text{ and } t_1 = 1.$$

(i) Write down the first three triangular numbers. **[1]**

(ii) Use the method of difference to show that

$$t_n = \frac{1}{2}n(n+1) \text{ for all } n \in \mathbb{Z}^+.$$ **[3]**

(iii) Find, in terms of N, the sum of the first N triangular numbers, giving your answer in a factorised form. **[2]**

[You may assume that $\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1)$.]

(b) In a certain experiment to study the growth of H2 bacteria in a cultured environment, the number of H2 bacteria (in thousands) exactly n days after the start of the experiment is denoted by a_n, where $a_n = 3^n + n - 1,\ n \in \mathbb{Z}^+$. Find

(i) the number of days after the start of the experiment when the population of the bacteria first exceeds 2014 thousands, **[2]**

(ii) the rate of growth of the H2 bacteria 2 days after the start of the experiment assuming that a_n is a continuous quantity. **[2]**

In another experiment, H3 bacteria is introduced together with the H2 bacteria. It is given that exactly 1 day after the start of the experiment, the number of H3 bacteria is 900 thousands and the number increases at a constant rate of 800 thousands per day thereafter. The number of H3 bacteria exactly n days after the start of the experiment is denoted by b_n.

(iii) Write down an expression for b_n in terms of n. **[1]**

(iv) By which day after the start of the experiment will the population of H2 bacteria exceed the population of H3 bacteria? **[2]**

Section B: Statistics [60 marks]

5 A school has 880 students. The principal wants to ask the student about the cleanliness of toilets in school. A survey is to be conducted with a random sample of 80 students from the school. Describe briefly how a systematic sample can be obtained. **[2]**

Explain why the sample obtained may not be representative of the school student population. Suggest a better sampling method. **[2]**

6 Two players, A and B, compete in a racquet match consisting of at most 3 sets. Each set is won by either A or B, and the match is won by the first person to win two sets. Player A has a probability of 0.6 of winning the first set. For each set after the first,

- the conditional probability that A wins that set, given that A won the preceding set, is p,
- the conditional probability that B wins that set, given that B won the preceding set, is 0.7.

Find, in terms of p, the probability that Player A wins the match. **[1]**

(i) The probability that Player B won the first set given that Player B had lost the match is 0.15. Find the value of p. **[2]**

(ii) Given instead that $p = 0.65$ and the probability that Player A's third win occurs in the nth match is 0.10866 correct to 5 significant figures, write down an equation for n, and solve it numerically. **[3]**

7 The bus leaves the bus stop near Adam's home at X minutes past 0745, where X is normally distributed with mean 25 minutes and standard deviation 3 minutes. Adam reaches the bus stop at Y minutes after 0745, where Y is normally distributed with mean 15 minutes and standard deviation 2 minutes. It is given that X and Y are independent. Find the probability that Adam misses the bus. **[3]**

The bus journey to school lasts W minutes, where W is normally distributed with mean 30 minutes and standard deviation $\sqrt{3}$ minutes. The random variable T denotes the number of minutes before 0830 at which the bus arrives at the school.

Express T in terms of W and X, and find the mean and variance of T. **[3]**

Find the probability that the bus would arrive at the school after 0830. **[2]**

8 **(i)** Sketch a scatter diagram that might be expected when x and y are related approximately as given in each of the cases (A) and (B) below. In each case your diagram should include 5 points, approximately equally spaced with respect to x, and with all x- and y-values positive. The letters a and b represent constants.

(A) $y = a + bx^2$ where a and b are positive,

(B) $y = a + \dfrac{b}{x}$ where a is positive and b is negative. **[2]**

A doctor assessed the extent of the lung damage of his 8 patients, who are smokers, using a scale from 0 to 100. The number of years each patient has been smoking and the lung damage rating are shown in the table.

Number of years patient has been smoking, x	15	22	25	31	36	39	42	48
Lung damage ratings, y	30	50	55	57	60	72	70	75

(ii) Draw the scatter diagram for these values, labelling the axes. **[1]**

(iii) Explain which of the two cases in part **(i)** is the most appropriate for modelling these values, and calculate the product moment correlation coefficient and the equation of a suitable regression line for this case. **[3]**

(iv) Use the case that you identified in part **(iii)** to find the estimate of the value of x for which $y = 64$ and comment on the validity of this estimated value. Explain why neither the regression line of x^2 on y nor the regression line $\dfrac{1}{x}$ on y should be used. **[3]**

9 A group of eight people consists of one pair of sisters, Amy and Betty, and six others. The group stands in a queue in random order. Find the number of ways of arranging if either Amy is first or Betty is last (or both). **[2]**

The group is then brought to a round table with seats numbered 1 to 12.

(i) Find the number of ways of arranging if the sisters are to sit together. **[3]**

(ii) Find the number of ways of arranging if the sisters are directly opposite each other. **[3]**

10 **(a)** A manufacturer claims that the mean length of steel pins produced by a machine is not more than 40 mm. A sample of 8 pins was taken and the length of each pin, x mm, is measured. The results are summarised by

$$\sum(x-40)=3.7\ , \quad \sum(x-40)^2=7.27.$$

(i) Stating a necessary assumption, test the manufacturer's claim at the 5% level of significance. **[5]**

(ii) Without carrying out any calculation, explain briefly if your conclusion would be different if a two-tail test at the 5% level of significance were to be used instead. **[1]**

(b) A random sample of 50 packets of cereal is weighed and the mass, x grams, is recorded. The results are summarised as $\bar{x}=148.4$ and $\sum(x-\bar{x})^2 = 2372$.

The manufacturer claims that the mean mass of a packet of cereal is k grams. Find the set of values for k, correct to 1 decimal place, given that his claim is rejected at the 4% level of significance. **[5]**

11 A garage sells GreatRun car tyres. The monthly demand for these tyres follows a Poisson distribution with mean 4. In a randomly chosen month, the garage has 5 such tyres.

(i) Find the probability that the garage is able to meet the demand. **[2]**

(ii) What is the most probable number of tyres sold? Show your workings clearly. **[2]**

(iii) What is the least number of tyres the garage should keep in stock at the beginning of the month in order that chances that the garage run out of stock is less than 0.001? **[2]**

It is given that the garage operates for 4 weeks each month.

(iv) Find the probability that the garage is able to sell more than 1 tyre weekly for more than half of the month. **[3]**

(v) Using a suitable approximation, show that in the first 10 years, the probability that the garage was not able to meet the monthly demands for at least 36 months given that the garage did not meet the monthly demands for more than 12 months, is 0.0154. **[5]**

——— END OF PAPER ———