



MERIDIAN JUNIOR COLLEGE  
JC2 Preliminary Examination  
Higher 2

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## H2 Mathematics

**9740/02**

### Paper 2

**22 September 2014**

**3 Hours**

Additional Materials:      Writing paper  
                                     List of Formulae (MF 15)

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### READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.

Write in dark blue or black pen on both sides of the paper.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

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This document consists of **7** printed pages.

**[Turn over**

### Section A: Pure Mathematics [40 marks]

- 1** Andrew decides to buy an apartment that costs \$350 000. He pays a deposit of 5% to the developer and finances the remaining amount by taking a bank loan on 1 January 2014. The bank charges interest at a rate of 0.25% per month at the beginning of each month, starting from 1 January 2014. In the middle of each month, he pays \$ $x$  to the bank. The repayment process is repeated every month until the loan is repaid in full.

Show that the outstanding amount owed at the end of the  $n$ th month is  $\$[332\,500(1.0025)^n - 400x(1.0025^n - 1)]$ . [4]

Deduce the least monthly repayment amount  $x$ , rounded off to the nearest dollar, required to pay off the entire loan by the end of the 30<sup>th</sup> year. [3]

When Andrew receives his first statement from the bank at the end of the first year, the outstanding loan amount is \$325 556. Explain why the amount shown on the statement is so near to the loan amount. [1]

- 2** The lines  $l_1$  and  $l_2$  have equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}, \lambda \in \mathbb{R} \quad \text{and} \quad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix}, \mu \in \mathbb{R}.$$

- (i) Determine whether the lines  $l_1$  and  $l_2$  are perpendicular. [1]

- (ii) Lines  $l_1$  and  $l_2$  intersect at the point  $P$ . Find the coordinates of  $P$ . [3]

The points  $A$  and  $B$  have position vectors  $\begin{pmatrix} 1 \\ 4 \\ 1 \end{pmatrix}$  and  $\begin{pmatrix} 5 \\ 2 \\ -1 \end{pmatrix}$  respectively. Line  $L$  is a perpendicular bisector of the line segment  $AB$ . The plane  $\pi$  contains all such  $L$ .

- (iii) Find an equation of  $\pi$  in the form  $\mathbf{r} \cdot \mathbf{n} = D$ . [3]

- (iv) Verify that plane  $\pi$  contains point  $P$ . [1]

- (v) Using your answers in parts (i) and (iv), give a geometrical interpretation of  $|\overrightarrow{PA} \times \overrightarrow{PB}|$ . [1]

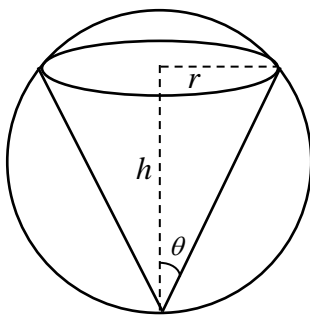
- 3 (i) Sketch on an Argand diagram the set of points representing all complex numbers  $z$  satisfying both relations:

$$-\frac{3\pi}{4} \leq \arg(z + 3i) \leq \frac{\pi}{2} \text{ and } |iz - 3| < |3i|. \quad [4]$$

- (ii) Find the least value of  $|z + 2 + 3i|$ . [2]

- (iii) Find the range of values of  $\arg(z + 2 + 3i)$ . [4]

- 4 The diagram shows a right inverted cone of radius  $r$ , height  $h$  and semi-vertical angle  $\theta$ , which is inscribed in a sphere of radius 1 unit.



Prove that  $r^2 = 2h - h^2$ . [1]

- (a) As  $r$  and  $h$  varies, determine the exact maximum volume of the cone. [5]

- (b) Show that  $h = 2\cos^2 \theta$ . The volume of the cone is increasing at a rate of  $6 \text{ cm}^3/\text{s}$  when  $h = \frac{3}{2}$ . Determine the rate of change of  $\theta$  at this instant, leaving your answer in the exact form. [7]

[The volume of a cone of radius  $r$  and height  $h$  is  $\frac{1}{3}\pi r^2 h$ .]

**Section B: Statistics [60 marks]**

- 5** A box contains 45 balls, which are distinctly labelled with integers 1 to 45. Seven balls are drawn randomly from the box without replacement, and the observed integers are recorded as ‘winning’ numbers.  
Andy chooses eight distinct integers randomly from the set of integers  $\{1, 2, 3, \dots, 44, 45\}$ .

- (i) Show that the probability that none of Andy’s chosen integers matches the ‘winning’ numbers is 0.227. [1]
- (ii) Hence find the probability that Andy’s chosen integers match exactly four of the ‘winning’ numbers, given that he has at least one winning number. [3]

- 6** A college has 1800 students. The table shows the number of boys and girls in each level.

	Boys	Girls
Year 1	490	380
Year 2	560	370

It is desired to sample 10% of the student population to find out their opinions of the food sold in the college canteen.

- (i) Describe how the sample could be chosen using systematic sampling. [2]
- (ii) State an alternative sampling method that would give a more representative sample and describe it briefly. [2]
- (iii) If the first 180 students who arrive at the college on a particular day are sampled instead, explain why this will not yield a random sample. [1]

- 7** Research is being carried out on how the height of a tree varies with its age. The data collected is given in the following table.

Age of tree ( $x$ years)	1	2	3	4	5	6	7	8
Height ( $y$ metres)	1.8	2.9	4.0	4.6	5.0	5.3	5.6	5.8

- (i) Draw the scatter diagram for these values, labeling the axes clearly. [2]
- (ii) Calculate the product moment correlation coefficient between  $x$  and  $y$ , and explain why its value does not necessarily mean that a linear model would be appropriate. [2]
- (iii) Explain why in this context a quadratic model would probably not be appropriate for long-term predictions. [1]
- (iv) The variable  $h$  is defined by  $h = \ln x$ . Calculate the equation of the appropriate regression line for variables  $h$  and  $y$ , and the value of the product moment correlation coefficient. [2]
- (v) Use a regression line to give the best estimate that you can of the age of the tree when the height is 5.2 metres. [2]

- 8** A manufacturer produces packets of nuts. The mean mass of contents in a packet is claimed to be 100 grams. The mass,  $x$  grams, of the contents of each packet in a random sample of 80 packets of nuts is measured and the results are summarized by

$$\sum (x - 50) = 3916 \qquad \sum (x - 50)^2 = 193\,375$$

- (i) Find unbiased estimates of the population mean and variance. [2]
- (ii) Test at, 4% level of significance, whether the population mean mass of the contents has been overstated. [5]
- (iii) Explain, in context of the question, the meaning of “at the 4% significance level”. [1]
- (iv) Another test is conducted at 4% level of significance using the same data as above. It is given that the hypotheses are as follows:

Null hypothesis : The population mean mass of contents is equal to 100 grams  
 Alternative hypothesis: The population mean mass of contents is not equal to 100 grams

Without carrying out the test, state with a reason, whether the conclusion of the test in part (ii) would remain the same. [1]

- 9** Tim takes part in a game show that requires him to answer a total of 4 questions. For each of the 4 questions, he is allowed to choose either an easy question or a difficult question to answer. He is more likely to choose an easy question over a difficult question. If Tim chooses an easy question, the probability that he answers it correctly is 0.97. If Tim chooses a difficult question, the probability that he answers it correctly is 0.20.

- (i) The probability that Tim answers 3 easy questions correctly and 1 difficult question incorrectly during the game show is 0.18. Show that the probability of him choosing an easy question is 0.921, correct to 3 significant figures. [2]

1 point is awarded for every easy question answered correctly, and 2 points are awarded for every difficult question answered correctly. Find the probability that

- (ii) Tim chooses 4 easy questions and scores 2 points during the game show. [3]
- (iii) Tim scores 4 points during the game show, given that he chooses at most 1 difficult question. [4]

- 10** The masses, in kilograms, for a randomly chosen pumpkin and a randomly chosen watermelon, are normally distributed with means 4.2 and 8.5, and standard deviations 1.6 and 2.2 respectively.

- (i) Find the range of values of  $k$  such that the average mass of five randomly chosen watermelons exceeds  $k$  kilograms occurs at most 10% of the time. [3]
- (ii) Find the probability that the total mass of five randomly chosen pumpkins differs from three times the average mass of five randomly chosen watermelons by more than 5 kg. [4]
- (iii) A fruit store has 70 batches, each consisting of  $n$  pumpkins. It is known that, on average, 3 in 20 pumpkins are rotten. Given that in the 70 batches, the probability of having an average of at least 4 rotten pumpkins per batch is more than 0.7. Determine the minimum value of  $n$ . [4]

- 11** (a) In a certain country, the mean number of people convicted of robbery in a week is 2. Use a Poisson distribution to find the probability that in a given month consisting of 4 weeks, at least 9 people are convicted of robbery. [3]

Use a suitable approximation to the Poisson distribution to find the probability that the number of people convicted of robbery in a year consisting of 52 weeks is not more than 90. [3]

Explain why the Poisson distribution may not be a good model for the number of people convicted of robbery in a year. [1]

- (b) In a certain country, the number of rainy days in June is observed. You may assume that the weather on each day is independent of the weather on any other days.

- (i) State, in context, an assumption needed for the number of rainy days in June to be well modelled by a binomial distribution. [1]

The probability that it rains on any given day in June is  $p$ .

- (ii) The probability that it rains at least once in a sample of 15 days in June is 0.96482. Write down an equation in terms of  $p$  and hence find the value of  $p$ , correct to 1 decimal place. [3]

- (iii) Given that  $p = 0.25$ , what is the most likely number of rainy days in June (consisting of 30 days)? [2]

**END OF PAPER**