NATIONAL JUNIOR COLLEGE
SENIOR HIGH 2 PRELIMINARY EXAMINATION
Higher 2

MATHEMATICS 9740/01

Paper 1

15 September 2014

3 hours

Additional Materials: Answer Paper
List of Formulae (MF15)
Cover Sheet

READ THESE INSTRUCTIONS FIRST

Write your name, registration number, subject tutorial group, on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in the brackets [ ] at the end of each question or part question.

This document consists of **6** printed pages, including the cover sheet.

National Junior College

1 It is given that the curve $y = ax^2 + bx + c$ passes through the point $A(-1, 20)$ and has a stationary point at $B(3, 4)$.

(i) Find the values of a, b and c. [3]

(ii) By completing the square for $ax^2 + bx + c$, or otherwise, describe a sequence of transformations that map $y = x^2$ onto $y = ax^2 + bx + c$. [2]

2 Find $\int \frac{x}{\left(1+4x^2\right)^2}\,\mathrm{d}x$. Hence find $\int \frac{4x^2}{\left(1+4x^2\right)^2}\,\mathrm{d}x$. [5]

3 The curve C has parametric equations $x = \frac{t}{t^2 - a}$, $y = t\mathrm{e}^{-t}$, where a is a positive real constant and $-\sqrt{a} < t \leq 0$.

(i) The tangent to the curve C at $t = 0$ is perpendicular to the line $4y - x = 0$.
Show that $a = 4$. [3]

Using the value of a in part **(i)**,

(ii) what can you say about the gradient of the curve C as $t \to -2$? [1]

(iii) sketch the curve C, including any points of intersection with the axes and the equation(s of any asymptotes. [2]

4 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$, where $\mathbf{a}$ and $\mathbf{b}$ are non-parallel vectors. The point M on OA extended is such that $OA : AM = 1:1$, and the point N on OB is such that $ON : NB = 1:2$. There exists a point P such that A, B and P are collinear and M, N and P are collinear.

(i) Find $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Show that the area of the quadrilateral $OAPN$ can be written as $k\left|\mathbf{a} \times \mathbf{b}\right|$, where k is a constant to be found. [3]

5 It is given that $f(x)=\dfrac{(x+k)^2}{x-k}$, $x\in\mathbb{R}$, $x\neq k$ and $k>0$.

(i) Use a non-calculator method to find the range of values of x for which f is increasing. [4]

(ii) Given that $k = 3$, sketch the curve $y = \mathrm{f}(x)$, labelling clearly the coordinates of any points of intersection with the axes, the coordinates of any stationary points, and the equation(s) of any asymptotes. [3]

6 The functions f and g are defined by

$$\mathrm{f}: x \mapsto \cos\left(\frac{\pi}{4}x\right) + 3.5, \text{ for } x\in\mathbb{R},\ 3\le x<4, \text{ and}$$
$$\mathrm{g}: x \mapsto |7-2x|, \text{ for } x\in\mathbb{R},\ 2<x\le 4.$$

(i) Find f^{-1}, stating its exact domain. [3]
(ii) Show that gf^{-1} exists. [1]
(iii) Sketch the graph of $y = \mathrm{g}(x)$ and find the range of gf^{-1}. [3]

7 The complex number z satisfies both $|z-4-3\mathrm{i}|\le 5$ and $\mathrm{Re}(z)\le 8$.

(i) On an Argand diagram, sketch the locus of z. [3]
(ii) Find the maximum value of $\arg(z+6-3\mathrm{i})$. Label the point corresponding to this maximum value on your diagram with the letter P and find the exact value of z in this case, leaving your answer in the form $x+\mathrm{i}y$. [5]

8 **(a)** In Marathon A, there are 20 participants. The prizes are awarded according to the following rule: the first prize is \$1000; the second prize is 4/5 of the first prize, the third prize is 4/5 of the second prize, and so on, till the 20th prize.

(i) What is the total amount of money that the organisers need to have for the prize fund? [2]
(ii) What is the assumption needed for your calculation in part **(i)** to be valid? [1]

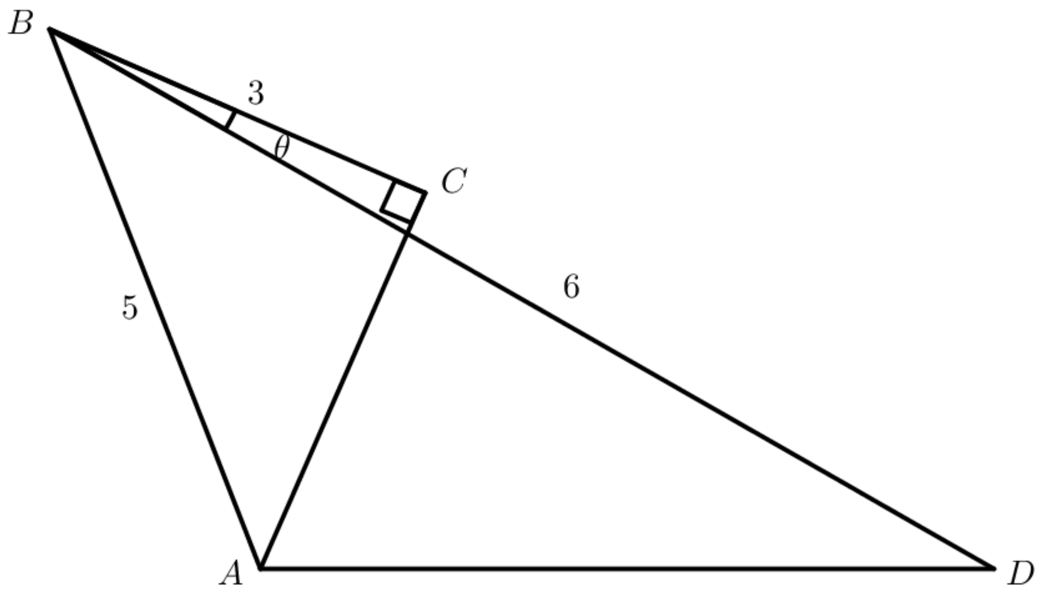
In Marathon B, the prizes are awarded according to the following rule: the last prize is \$15, the second last prize is \$185 more than the last prize, the third last prize is \$185 more than the second last prize, and so on, till the first prize. Determine the maximum number of prizes to be awarded if the organisers have only \$100 000 to sponsor the prize fund.

[2]

(b) The sum of the first n terms of a series is given by $10-\dfrac{2^{n+1}}{5^{n-1}}$. Show that the series is a geometric series and state the value of the common ratio. [4]

[Turn over

9 **(a)**



In the diagram above, $AB = 5$, $BC = 3$, $BD = 6$, angle $ACB = \dfrac{\pi}{2}$ radians and angle $CBD = \theta$ radians. Given that θ is a sufficiently small angle, show that

$$AD \approx \left(25 - 48\theta + 18\theta^2\right)^{\frac{1}{2}} \approx 5 - \frac{24}{5}\theta + k\theta^2,$$

for a constant k to be determined. [6]

(b) Given that the Maclaurin series expansion of $e^{\sin^{-1} nx}$ is $1 + 2x + bx^2$, find the values of the real constants b and n. [4]

10 A sequence $u_1, u_2, u_3, \ldots$ is such that $u_1 = 1$ and $u_{n+1} = u_n + \dfrac{6}{(n+2)(n+3)}$, for $n \geq 1$.

(i) Prove by mathematical induction that $u_n = \dfrac{3n}{n+2}$. [4]

(ii) Find $\displaystyle\sum_{n=1}^{N} \frac{6}{(n+2)(n+3)}$. [2]

(iii) Hence find the exact value of $\displaystyle\sum_{n=10}^{\infty} \frac{3}{(n+2)(n+3)}$. [3]

(iv) Using the result obtained in part **(ii)**, find $\displaystyle\sum_{j=6}^{N} \frac{6}{(j-2)(j-3)}$ in terms of N. [2]

[Turn over

11 **(a)** Show that the differential equation $\frac{\mathrm{d}y}{\mathrm{d}x}+2y=(x+1)\mathrm{e}^{x^2}$ may be reduced by the substitution $z=\mathrm{e}^{2x}y$ to $\frac{\mathrm{d}z}{\mathrm{d}x}=(x+1)\mathrm{e}^{x^2+2x}$. Hence, find the general solution for y in terms of x. [5]

(b) A freshly brewed cup of coffee has an initial temperature 95°C. It is placed in a room where the temperature is a constant at 20°C. As the coffee cools down, the rate of decrease of its temperature θ °C after time t minutes is proportional to the temperature difference (θ – 20)°C. It is cooling at a rate of 1°C per minute when its temperature is 70°C.

(i) By setting up a differential equation, find the rate of cooling of the coffee when its temperature is 40°C and show that $\theta=20+75\mathrm{e}^{-0.02t}$. [6]

(ii) Comment on whether the model can be regarded as a good model of the situation in the real world. [1]

12 The curve C_1 has equation $\frac{x^2}{4}+y^2=1$. The curve C_2 has equation $\frac{x^2}{4}-y^2=1$.

(i) Sketch C_1 and C_2 on the same diagram, labelling clearly the exact coordinates of the point(s) of intersection with the axes and the equation(s) of the asymptote(s), if any. [4]

(ii) Find the volume of revolution when the region bounded by C_1, C_2 and the line $y=1$ for $x\geq 0$ is rotated completely about the x-axis. Give your answer correct to 4 decimal places. [4]

(iii) By using a substitution of the form $x=a\cos\theta$, where a is a positive constant and $0\leq\theta\leq\frac{\pi}{2}$, find the exact area bounded by C_1, the positive x-axis and the line $x = 1$. [6]

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[Turn over

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Candidate Name: ________________________

Registration No.: ________________________

Subject Class: ___________________________

Subject Tutor: __________________________

Cover Sheet

INSTRUCTIONS TO CANDIDATES

Write your name, registration number, subject tutorial group, subject tutor's name and calculator model in the spaces provided on the cover sheet and attached it on top of your answer paper.

Circle the questions you have attempted and arrange your answers in **NUMERICAL ORDER**.

Write your calculator's model number(s) in the box below.

Scientific Calculator Model:

Graphing Calculator Model:

For official use

Question No.	Marks Obtained	TOTAL MARKS
1		5
2		5
3		6
4		6
5		7
6		7
7		8
8		9
9		10
10		11
11		12
12		14
Presentation	–1 / –2	
TOTAL		100
GRADE		