

**NATIONAL JUNIOR COLLEGE**  
**SENIOR HIGH 2 PRELIMINARY EXAMINATION**  
**Higher 2**

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**MATHEMATICS**

**9740/02**

Paper 2

**16 September 2014**

**3 hours**

Additional Materials:      Answer Paper  
                                     List of Formulae (MF15)  
                                     Cover Sheet

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**READ THESE INSTRUCTIONS FIRST**

Write your name, registration number, subject tutorial group, on all the work you hand in.  
Write in dark blue or black pen on both sides of the paper.  
You may use a soft pencil for diagrams or graphs.  
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.  
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  
You are expected to use an approved graphing calculator.  
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.  
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.  
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.  
The number of marks is given in the brackets [ ] at the end of each question or part question.

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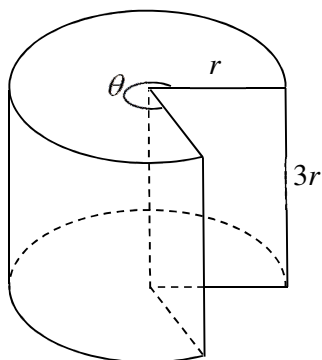
This document consists of **7** printed pages, including the cover sheet.



**National Junior College**

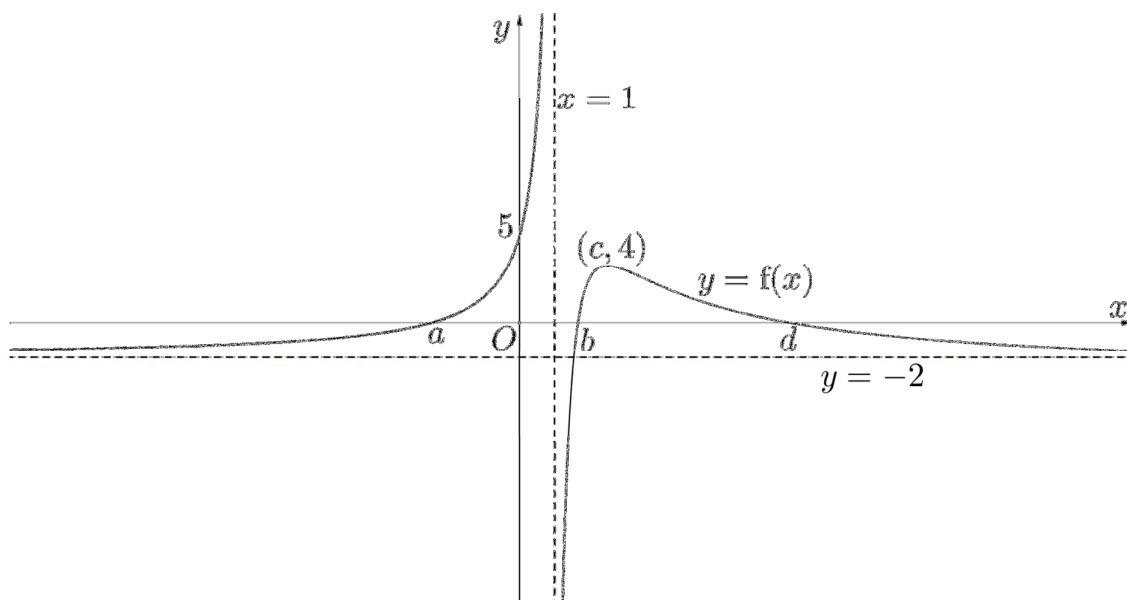
# Section A: Pure Mathematics [40 marks]

1



A solid has height  $3r$  cm and a base that is made up of a circular sector with radius  $r$  cm. The sector subtends an angle of  $\theta$  radians, where  $0 < \theta < 2\pi$ , at the centre of the circle as shown in the diagram. Given that the total surface area of the container is  $10 \text{ cm}^2$ , find the exact values of  $r$  and  $\theta$  such that the volume of the container is a maximum. [8]

2



The diagram shows the curve with equation  $y = f(x)$ . The curve crosses the  $x$ -axis at  $x = a$ ,  $x = b$  and  $x = d$ , and has a turning point at  $(c, 4)$ , where  $a, b, c$  and  $d$  are real constants. The lines  $x = 1$  and  $y = -2$  are asymptotes to the curve.

Sketch the graphs of

(a)  $y^2 = f(x)$ , [3]

(b)  $y = \frac{1}{f(x)}$ , and [4]

(c)  $y = f'(x)$  [3]

on separate diagrams. Label clearly any points where the graphs cross the axes, the coordinates of any turning points and the equations of any asymptotes.

- 3 (a) Given that

$$\left| \frac{2i - z^*}{z} - 1 \right|^2 - z = i,$$

find  $z$  in the form  $x + iy$ . [4]

- (b) Let  $p = -\sqrt{3} + i$  and  $q = -4i$ .

(i) Write down  $p$  and  $q$  exactly in the form  $re^{i\theta}$ , where  $r > 0$  and  $-\pi < \theta \leq \pi$ . [2]

(ii) Find the exact value of  $\frac{p^{10}}{q^5} + \frac{q^5}{p^{10}}$ . [4]

- 4 The plane  $p_1$  has equation  $x - 3y + z = 4$ . Relative to an origin  $O$ , the point  $C$  has position

vector  $\begin{pmatrix} -3 \\ 5 \\ 0 \end{pmatrix}$ . The foot of the perpendicular from  $C$  to  $p_1$  is  $M$ .

(i) Find the position vector of  $M$ . [4]

(ii) Find the coordinates of the point  $Q$  which is the mirror image of  $C$  in  $p_1$ . [2]

Another plane  $p_2$  passes through the point  $C$  and is perpendicular to the  $y$ -axis. The planes  $p_1$  and  $p_2$  intersect at a line  $l$ .

(iii) Find a vector equation of  $l$ . [2]

(iv) Find a cartesian equation of  $p_3$ , the mirror image of  $p_2$  in  $p_1$ . [4]

### Section B: Statistics [60 marks]

- 5 (a) 8 students are attending a Mathematics lecture in a lecture theatre. Each student occupies exactly one seat. Find the number of ways that all 8 students can be seated in a particular row of 10 seats if there are no restrictions. [2]

(b) The group of 8 students includes a boy named Derrick, as well as 3 close friends, Amy, Alicia and Andrea. In another lecture, the 8 students occupy another row comprising exactly 8 seats. Find the number of ways that the 8 students can be seated in this row if Amy, Alicia and Andrea are to be seated together and none of them are seated next to Derrick. [3]

- 6 The admissions office of a particular university wishes to survey 50 students from the graduating cohort of a particular junior college, which comprises exactly 550 students, about which university courses and career paths they are interested to pursue. Describe, in context of the question, how the admissions office can obtain the sample by using

(a) systematic sampling, [2]

(b) quota sampling. [2]

Explain briefly, in the context of your answers to parts (a) and (b), a reason why quota sampling may be more suitable than systematic sampling for the purpose of the survey. [1]

- 7 In Novel Junior College, students take attendance by tapping their fingers at a biometric fingerprint sensor before morning assembly for any school day. At any of the fingerprint sensors, no two students can tap their fingers at the same instant in time. Also, no student taps his/her finger more than once on any school day.

(a) State two assumptions needed for the number of students who tap their fingers at a particular fingerprint sensor during a randomly chosen one-minute time interval to be well modelled by a Poisson distribution. [2]

Assume that the number of students who tap their fingers at a particular fingerprint sensor during a one-minute time interval has the distribution  $Po(4.2)$ .

(b) Find the probability that at least 3 students tap their fingers at the fingerprint sensor in a one-minute period. [1]

(c) It is given that the probability that exactly 10 students tap their fingers at the fingerprint sensor in a  $t$ -second period is 0.1. Find an equation for  $t$ . Hence find the possible values of  $t$ , giving your answers correct to the nearest whole number. [3]

- 8 The random variable  $X$  has the distribution  $B\left(64, \frac{5}{8}\right)$ .

(a) Find

(i)  $P(X = 34 \text{ or } 35)$ , [2]

(ii)  $P(X = 34 \text{ or } 35 \mid X \leq E(X))$ . [2]

(b) Using a suitable approximation, find  $P(30 < X \leq 45)$ . [3]

- 9 (a) Mrs Wang likes to bake cupcakes. On one fine day, she bakes 15 chocolate-flavoured and 25 blueberry-flavoured cupcakes. After baking, she gives away 8 randomly chosen cupcakes. Find the probability that exactly 3 chocolate-flavoured cupcakes are given away. [2]
- (b) Studies have shown that flu vaccines reduce the chance of one getting the flu. During a seasonal flu period, the probability that a person gets a flu vaccine is 0.85. Also, the probability that a person who had the flu vaccine gets the flu is 0.23, and the probability that a person who did not get the flu vaccine gets the flu is 0.91. Find the probability that
- (i) one gets the flu. [2]
- (ii) out of two randomly chosen people who got the flu, exactly one of them took the flu vaccine. [3]
- 10 The weight in kg of a whole chicken sold in “Shop & Pay” is a random variable with a distribution  $N(2.2, 0.5^2)$ .
- (i) The mean weight of  $n$  randomly chosen chickens is denoted by  $\bar{C}$  kg. Given that  $P(\bar{C} > 2.35) = 0.0502$ , find the value of  $n$ . [3]

The weight in kg of a whole turkey sold in “Shop & Pay” is a random variable with a distribution  $N(11, 2.1^2)$ . Chickens are sold at \$3 per kg and turkeys at \$5 per kg.

- (ii) Calculate the probability that the total price of 17 randomly chosen chickens is more than twice the price of a randomly chosen turkey. State clearly the mean and variance of any normal distribution you use in your calculation. [4]
- (iii) State an assumption needed for your calculation in part (ii). [1]

- 11** A cookie manufacturing company claims that the mean number of calories of a mini-sized packet of its chocolate chip cookies is 100. The number of calories of a mini-sized packet of chocolate chip cookies is denoted by  $X$ . The number of calories for a random sample of 10 such mini-size packets of cookies are summarised by

$$\sum(x - 100) = 19, \quad \sum(x - 100)^2 = 159.$$

- (i) Calculate unbiased estimates of the mean and variance of  $X$ . [2]

A nutrition lab wants to know whether the above sample indicates that the company is underestimating the mean number of calories claimed for a mini-sized packet of its chocolate chip cookies. A test is carried out at the  $\alpha\%$  significance level.

- (ii) State a necessary assumption to carry out the test, and the appropriate hypotheses for the test. Define any symbols you use. [2]  
 (iii) Carry out the test for  $\alpha = 5$ . [3]  
 (iv) Explain, in the context of the question, the meaning of “at the  $\alpha\%$  significance level”. [1]

Suppose now the population variance of  $X$  is known to be 13. With the assumption made in (ii) and for  $\alpha = 2$ , use an algebraic method to calculate the set of values of the sample mean for another random sample of 10 mini-sized packets of chocolate chip cookies such that it is concluded that the mean number of calories for a mini-sized packet is not equal to 100. [3]

- 12** The table below gives the age and weight for 10 broiler chicks.

Age (Days), $x$	18	1	10	14	13	15	17	12	16	20
Weight(g), $y$	108	44	66	84	80	92	100	76	104	118

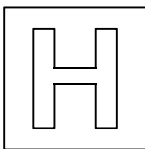
- (i) Sketch the scatter diagram. [1]  
 (ii) Determine the value of the product moment correlation coefficient,  $r$ , between  $y$  and  $x$ . [1]  
 (iii) Explain why the value of  $r$  does not necessarily signify that a linear model is a good fit. [1]  
 (iv) State whether it is appropriate to use the  $x$  on  $y$  line or the  $y$  on  $x$  line to estimate  $x$  when  $y = 55$ . Justify why. [2]  
 (v) Determine which of the following is the best model for this set of data, justifying your choice clearly.

$$(A) \ y = ax + b \quad (B) \ y = cx^2 + d \quad (C) \ y = e\sqrt{x} + f,$$

where  $a, b, c, d, e$  and  $f$  are positive constants [2]

- (vi) Find the equation of the least-squares regression line of your selected best model in part (v). Use your equation to estimate the value of  $y$  when  $x = 8$ . Comment on the reliability of the estimation. [4]

— End of Paper —



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**16 September 2014**

**Candidate Name:** \_\_\_\_\_

**Subject Class:** \_\_\_\_\_

**Registration No.:** \_\_\_\_\_

**Subject Tutor:** \_\_\_\_\_



**over Sheet**

**INSTRUCTIONS TO CANDIDATES**

Write your name, registration number, subject tutorial group, subject tutor's name and calculator model in the spaces provided on the cover sheet and attached it on top of your answer paper.

Circle the questions you have attempted and arrange your answers in **NUMERICAL ORDER**.

Write your calculator's model number(s) in the box below.

Scientific Calculator Model:

Graphing Calculator Model:

*For official use*

Question No.	Marks Obtained	TOTAL MARKS
1		8
2		10
3		10
4		12
5		5
6		5
7		6
8		7
9		7
10		8
11		11
12		11
Presentation	-1 / -2	
TOTAL		100
GRADE		