JURONG JUNIOR COLLEGE

Preliminary Examinations

MATHEMATICS
Higher 2

Paper 2

9740/02

17 September 2014

3 hours

Additional materials: Answer Paper
Cover Page
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together, with the cover page in front.

This document consists of **6** printed pages.

Section A: Pure Mathematics [40 marks]

1 **(a)** Given that $e^{y+x} = \cos x$, show that $\dfrac{dy}{dx} + \tan x + 1 = 0$.

(i) By further differentiation of this result obtain the Maclaurin's series for y in terms of x, up to and including the term in x^2. [3]

(ii) Let the result in **(i)** be $h(x)$. Find the set of values of x for which $h(x)$ is within ± 0.2 of the value of y. [2]

(b) Expand, in ascending powers of x, $\left(a - \dfrac{x}{3}\right)^n$ where $a \in \square$ and n is a non-positive integer, up to and including the term in x^2. [2]

It is given that the coefficient of x is four times the coefficient of x^2 and the constant in the expansion is $\dfrac{1}{4}$. Find a and n. [3]

2 **(a)** Find the general solution of the differential equation

$$\frac{d^2 y}{dx^2} = a e^{-2x}, \text{ where } a \text{ is a constant.} \quad [2]$$

(b) Mr Tan borrowed \$10 000 from a bank and at time t, the sum of money he owed the bank is denoted by x thousand dollars. The sum of money he owed increases, due to interest, at a rate proportional to the sum of money owed. Money is also repaid at a constant rate p. When $x = 12$, the interest and repayment balance. Taking both x and t as continuous variables, show that for $x > 0$,

$$\frac{dx}{dt} = \frac{p}{12}(x - 12). \quad [2]$$

(i) Find x in terms of t and p. [4]

(ii) Find the time T that it will take Mr Tan to repay the loan, leaving your answer in terms of p. [1]

(iii) Sketch the graph of x against t. [1]

3 The plane π_1 has equation, $\mathbf{r}.\begin{pmatrix}\alpha\\ \beta\\ 0\end{pmatrix}=-30$, where α and β are positive constants, and contains the point A with coordinates $(-10,\ 0,\ 5)$.

(i) Given that the perpendicular distance from the origin O to the plane π_1 is 6, find α and β. [5]

Another plane π_2 has equation $\mathbf{r}.\begin{pmatrix}-1\\ 1\\ 2\end{pmatrix}=4$.

(ii) Find the acute angle between the line OA and the plane π_2. [2]

(iii) Find a cartesian equation of the plane π_3 which contains the line OA and is perpendicular to the plane π_2. [3]

4 The complex numbers p and q are given by

$$p=1-\mathrm{i} \text{ and } q=1+\sqrt{3}\,\mathrm{i}.$$

(i) Two other complex numbers r and s are given by

$$r=k+\mathrm{i} \text{ and } s=\frac{p^2r}{q^3}, \text{ where } k \text{ is a real number.}$$

Given that $|s|=\frac{1}{2}$ and $\arg(s)=-\frac{2}{3}\pi$, find the value of k. [5]

(ii) Solve the equation

$$z^3=4q. \qquad [2]$$

Hence find the roots of the equation

$$w^3=4\left(\sqrt{3}-\mathrm{i}\right). \qquad [3]$$

[Express all answers in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $-\pi<\theta\le\pi$, giving r and θ in exact form.]

Section B: Statistics [60 marks]

5 A company wants to interview residents from a town about the brand of shoes that they prefer. An interviewer stationed himself at the town centre and interviewed 100 residents of different age groups.

(i) Name the sampling method used by the interviewer and state one disadvantage of this method in the context of the question. [2]

The company would now like to find out more about the preferences of the youths in a youth club with members aged 18 to 35. The table below shows the number of male and female members of each age group.

	18 - 25 year old	**26 - 35 year old**
Male	75	36
Female	99	40

(ii) Explain how a stratified sample of size 100 could be obtained. [2]

6 The random variable X has the distribution $\mathrm{N}\left(\mu,\sigma^2\right)$. Given that X_1 and X_2 are two independent observations of X and $\mathrm{P}\left(2X_1 < X_2\right) = \mathrm{P}\left(X_1 + X_2 > 2a\right) = 0.8$, where a is a constant, find μ in terms of a. [7]

7 For events *A* and *B*, it is given that $\mathrm{P}(A)=0.6$, $\mathrm{P}\left(B \mid A'\right)=0.75$ and $\mathrm{P}\left(A \mid B\right)=0.4$.

Find

(i) $\mathrm{P}\left(A' \cap B'\right)$, [3]

(ii) $\mathrm{P}(B)$. [3]

Justifying your conclusion, determine whether *A* and *B* are independent. [1]

8 Four letters of the word ABSENCE are selected and arranged to form four-letter code words. Show that the number of four-letter code words that can be formed without restrictions is 480. [3]

(i) Find the probability that a randomly chosen four-letter code word contains distinct letters. [2]

(ii) A four-letter code word containing distinct letters is randomly chosen. Find the probability that it does not contain any vowels (A, E). [3]

9 A random sample of nine pairs of values of *x* and *y* are given in the table.

x	2.5	2.0	3.0	3.5	5.0	4.0	5.3	7.5	6.0
y	3.20	3.40	3.00	2.86	2.61	2.75	2.57	k	2.55

(i) The equation of the regression line of *y* on *x* is $y=-0.175x+3.57$. Show that $k=2.40$. [3]

(ii) Draw a scatter diagram for this set of data and obtain the product moment correlation coefficient. Comment on the suitability of the linear model. [3]

(iii) State, with a reason, which of the following would be an appropriate model to represent the above data. The letters *a, b, c, d, e* and *f* represent constants.

A. $y=a+bx^2$, where *a* is positive and *b* is negative,

B. $y=c+d\ln x$, where *c* is negative and *d* is positive,

C. $y=e+\dfrac{f}{x}$, where *e* is positive and *f* is positive. [1]

(iv) It is required to estimate the value of *y* for which *x* = 8.0. Find the equation of a suitable regression line, and use it to find the required estimate. Comment on the reliability of your estimation. [3]

[Turn over

10 The number of packets of cereal sold in a supermarket is a normally distributed random variable. Over a long period it is known that the mean number of packets of cereal sold per day is 124.5. Following a promotional campaign on television advertisement, the daily sales, x packets, is measured for a random sample of 12 days. The results are summarized as follows:

$$\sum x = 1620 \quad \text{and} \quad \sum x^2 = 221175.$$

(i) Find unbiased estimates of the population mean and variance. [2]

(ii) Test, at the 1% level of significance, whether the mean number of packets of cereal sold per day has changed. [4]

After some modifications to the promotional campaign, a new sample of 12 days is taken and the population standard deviation is known to be 13. The owner of the supermarket claims that the mean daily sales has increased from 124.5. Find the range of values of the mean daily sales of the new sample if the owner's claim is to be accepted at the 1% level of significance. [4]

Explain what you understand by the expression 'at the 1% level of significance' in the context of this question. [1]

11 A small hospital receives emergency admissions randomly at an average rate of 3 per day.

(i) Find the probability that there will be a total of 3 emergency admissions in two randomly chosen consecutive days. [2]

(ii) Find the length of time, to the nearest hour, for which the probability that no admissions are received is 0.2. [3]

(iii) Calculate the probability that there will be more than 4 emergency admissions on a randomly chosen day. [1]

A random sample of 50 days is taken.

(iv) Using a suitable approximation, find the probability that, the hospital will have at most 4 emergency admissions in less than 40 of these days. [3]

The staff of the hospital emergency unit observes that, on average, the proportion of the admitted patients who require surgery is 40%.

(v) State the mean and variance of the number of patients who require surgery in a random sample of 15 patients. In a study conducted over a period of time, 50 such samples of fifteen patients are randomly selected. Find the probability that the mean number of admitted patients that require surgery is more than 5.5. [4]