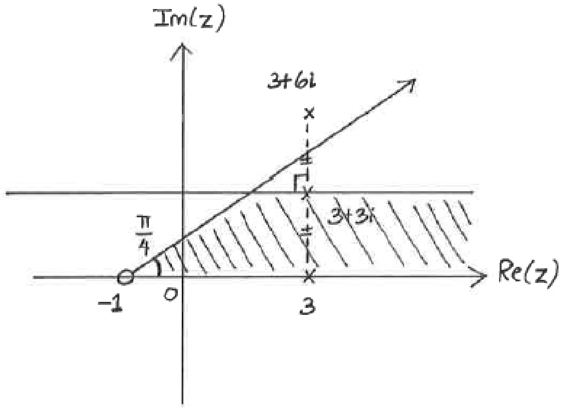
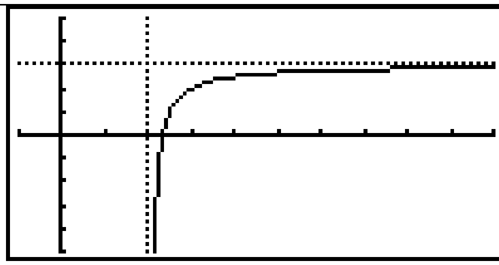
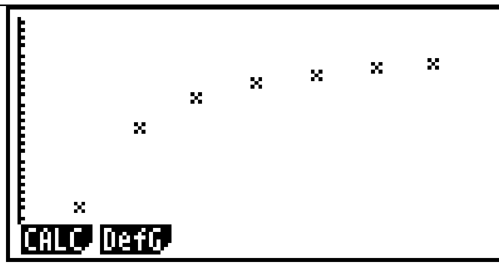
2014 MI H2 Mathematics Paper 2 Answer

1 (i)	$\frac{-x^2+4x+2}{x+4} \geq 1$ $\frac{x^2-3x+2}{x+4} \leq 0$ $\frac{(x-1)(x-2)}{x+4} \leq 0$ $x < -4$ or $1 \leq x \leq 2$
1 (ii)	$\frac{-2\sin^2\theta+4\sin\theta+1}{\sin\theta+2} \geq 1$ $\frac{-4\sin^2\theta+8\sin\theta+2}{2\sin\theta+4} \geq 1$ $\frac{-(2\sin\theta)^2+4(2\sin\theta)+2}{(2\sin\theta)+4} \geq 1$ $2\sin\theta < -4$ or $1 \leq 2\sin\theta \leq 2$ No solutions $\qquad \frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$
2	$10000 = x^2 h$ $h = \frac{10000}{x^2}$ Let C be the total cost in dollars. $C = 5x^2 + 2(4xh)$ $C = 5x^2 + \frac{80000}{x}$ $\frac{\mathrm{d}C}{\mathrm{d}x} = 10x - \frac{80000}{x^2}$ Let $\frac{\mathrm{d}C}{\mathrm{d}x} = 0$, $x = 20$ $\frac{\mathrm{d}^2C}{\mathrm{d}x^2} = 10 + \frac{160000}{x^3}$ $\left.\frac{\mathrm{d}^2C}{\mathrm{d}x^2}\right\|_{x=20} = 30 > 0$ Hence, C is minimum when $x = 20$. $C_{\min} = 6000$

3 (i)	
3 (ii)	$\sqrt{[2-(-2)]^2+(3-7)^2}$ $=\sqrt{32}$
3 (iii)	$-\tan^{-1}3<\arg(z-(-2+3\mathrm{i}))\le 0$
4 (i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{2t}$ Equation of tangent at the point with parameter p: $y-\left(2-\frac{1}{2p}\right)=\frac{1}{2p}(x-\ln p)$ $y=\frac{x}{2p}-\frac{\ln p}{2p}-\frac{1}{2p}+2$
4 (ii)	Equation of tangent at the A (i.e. $p=1$): $y=\frac{x}{2}+\frac{3}{2}$ When $y=0$, $x=-3$ $T(-3,0)$ Equation of normal at A: $y=-2x+\frac{3}{2}$ When $y=0$, $x=\frac{3}{4}$ $N\left(\frac{3}{4},0\right)$ Area of $\Delta ANT=\frac{45}{16}$

5 (a) (i)	 Asymptotes: $x=2$, $y=3$ Axial intercepts: $\left(\frac{7}{3},0\right)$
5 (a) (ii)	$\mathrm{f}^{-1}(x)=2-\frac{1}{x-3},\ \ x<3,\ \ x\in\mathbb{R}$
5 (a) (iii)	$R_g=\left(\frac{1}{2-a}+3,3\right)$ Let $\frac{1}{2-a}+3=2$ $a=3$
5 (a) (iv)	$\mathrm{fg}(x)=\frac{2-x}{x-3}+3,\ \ x>3,\ \ x\in\mathbb{R}$ $R_{\mathrm{fg}}=(-\infty,2)$
5 (b)	$\mathrm{f}(x)=\frac{1}{2}\left(1-\frac{x}{2}\right)^{-1}+3$ $=\frac{1}{2}\left[1+(-1)\left(-\frac{x}{2}\right)+\frac{(-1)(-2)}{2!}\left(-\frac{x}{2}\right)^2\right.$ $\left.+\frac{(-1)(-2)(-3)}{3!}\left(-\frac{x}{2}\right)^3+\cdots\right]+3$ $=\frac{7}{2}+\frac{x}{4}+\frac{x^2}{8}+\frac{x^3}{16}+\cdots$
6 (i)	A random sample is a sample where every employee has an equal chance of being selected and each employee is selected independently of each other.
6 (ii)	Assign each employee a number from 1 to 1000. Use a random number generator to generate 100 distinct numbers between 1 and 1000 (inclusive). Select the employees with the randomly generated numbers.
6 (iii)	Stratified sampling. The number of employees in the sample is proportional to the number of employees that work in the different shifts and hence the sample will give a better representation of all

	the employees.
7 (i)	Let S_n be the random variable for the number of students who stop to watch the concert in an n-minute interval. $S_3 \sim \text{Po}(18)$ $P(S_3 \geq 15) = 1 - P(S_3 \leq 14) = 0.79192$ $= 0.792$ (3 s.f.)
7 (ii)	Let L_t be the random variable for the number of students who leave the concert venue in a t-minute interval. $L_t \sim \text{Po}(3t)$ $\text{P}(L_t \geq 1) = 0.6$ $\text{P}(L_t = 0) = 0.4$ $\text{e}^{-3t} = 0.4$ $t = 0.30543$ (5 d.p.)
7 (iii)	$S_5 \sim \text{Po}(30)$ $\qquad$ $L_5 \sim \text{Po}(15)$ Since $30 > 10$, $S_5 \sim \text{N}(30, 30)$ approximately Since $15 > 10$, $L_5 \sim \text{N}(15, 15)$ approximately $S_5 - L_5 \sim \text{N}(15, 45)$ $\text{P}(10 \leq S_5 - L_t < 20)$ $\overset{c.c.}{=} \text{P}(9.5 \leq S_5 - L_5 < 19.5)$ $= 0.543$ (3 s.f.)
8 (i)	$\bar{x} = 10.2$ $s^2 = 0.031667 = 0.0317$ (3 s.f.)
8 (ii)	Assume X follows a normal distribution. $H_0 : \mu = 10$ $H_1 : \mu \neq 10$ Perform a 2-tailed t-test. Test statistic, $t = \dfrac{\bar{X} - \mu}{s / \sqrt{n}} \sim t(12)$ where $s = \sqrt{0.031667}$, $n = 13$ $t = 4.0523$ p-value $= 0.0016038 < 0.05 = \alpha$ Reject H_0. There is sufficient evidence at the 5% significance level to conclude that the mean amount of rice in each packet is no longer 10 kg.
8 (iii)	$\alpha = 0.160$ (3 s.f.)
9	$[\text{P}(-1 < X < 1)][\text{P}(-1 < Y < 1)]$

(i)	$= 0.099988$ (5 s.f.)
9 (ii)	$[\mathrm{P}(-5 < X < 5)][\mathrm{P}(-5 < Y < 5)] - 0.099988$ $= 0.893187 - 0.099988$ $= 0.793199 = 0.793$ (3 s.f.)
9 (iii)	$\frac{0.793199}{0.893187}$ $= 0.888$ (3 s.f.)
9 (iv)	$(0.793199)^2 + 2(0.099988)(1 - 0.893187)$ $= 0.651$ (3 s.f.)
10 (i)	
10 (ii)	Model (A): $r = 0.867$ Model (B): $r = -0.999$ For model (B), $\lvert r \rvert$ is closer to 1 compared to model (A). Hence, model (B) is more appropriate. OR As t increases, the values of h increase at a decreasing rate. Hence, model (B) is more appropriate.
10 (iii)	Regression line: $h = -20.437\frac{1}{t} + 25.642$ When $h = 22$, $t = 5.61$ (3 s.f.)
10 (iv)	a remains unchanged. b will increase by k.
11 (i)	$\left(\frac{26}{36}\right)^4 = 0.272$ (3 s.f.)
11 (ii)	$\frac{9+8+7+6+5+4+3+2+1}{36^2} = 0.0347$ (3 s.f.)
11 (iii)	$\frac{(26)(25)(24)(10)({}^4C_1)}{36^4} = 0.372$ (3 s.f.)

11 (iv)	$\dfrac{(26)(10)(9)({}^4C_2)+(10)(26)(25)({}^4C_2)}{36^4}$ $=0.0316$ (3 s.f.)
12 (i)	Let S be the random variable for the lifetime of a Standard battery in hours. $S \sim \mathrm{N}(300, 30^2)$ $\mathrm{P}(S > 310) = 0.369$ (3 s.f.)
12 (ii)	Let L be the random variable for the lifetime of a Long Lasting battery in hours. $L \sim \mathrm{N}(500, 50^2)$ $S_1 + \cdots + S_{10} - 5L \sim \mathrm{N}(500, 71500)$ $\mathrm{P}(S_1 + \cdots + S_{10} - 5L < 0) = 0.0307$ (3 s.f.)
12 (iii)	Let T be the random variable for the number of torchlights out of 10 that are in working condition. $T \sim \mathrm{B}(10, 0.97)$ $\mathrm{P}(T \geq 9) = 1 - \mathrm{P}(T \leq 8)$ $= 0.965493 = 0.965$ (3 s.f.)
12 (iv)	Let U be the random variable for the number of boxes out of 15 that have at least 9 torchlights that are in working condition. $U \sim \mathrm{B}(15, 0.965493)$ $\mathrm{P}(12 < U < 15) = \mathrm{P}(U \leq 14) - \mathrm{P}(U \leq 12)$ $= 0.396$ (3 s.f.)
12 (v)	Let V be the random variable for the number of torchlights out of 100 that are spoilt. $V \sim \mathrm{B}(100, 0.03)$ $n = 100 \geq 50$ is large $np = 3 < 5$ $V \sim \mathrm{Po}(3)$ approximately $\mathrm{P}(V \geq 5) = 1 - \mathrm{P}(V \leq 4) = 0.185$ (3 s.f.)
12 (vi)	$S + L \sim \mathrm{N}(800, 3400)$ $(0.97)[\mathrm{P}(S + L > 750)] = 0.780$ (3 s.f.)