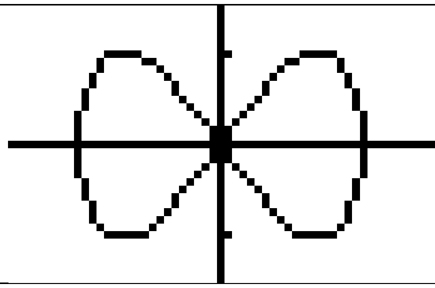
1i	$\begin{pmatrix}0\\0\\1\end{pmatrix}+\lambda\begin{pmatrix}-2\\1\\-1\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}+\mu\begin{pmatrix}1\\-2\\1\end{pmatrix}$ Since $\begin{pmatrix}-2\\1\\-1\end{pmatrix}\neq m\begin{pmatrix}1\\-2\\1\end{pmatrix}$ for m$\in\mathbb{R}$, the lines are not parallel. $-2\lambda=1+\mu$ --- (1) $\lambda=1-2\mu$ ----- (2) $1-\lambda=\mu$ ------- (3) Solving (1) and (2): $\lambda=-1,\ \mu=1$ when $\lambda=1,\ \mu=0,\ (1):\ \text{LHS}=-2\neq1=\text{RHS}$ Therefore the lines are skewed.
1ii	$\text{angle}=\cos^{-1}\frac{\left\lvert\begin{pmatrix}-2\\1\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}\right\rvert}{\sqrt{6}\sqrt{6}}$ $=33.6°$
2i	$\sin(A+B)+\sin(A-B)$ $=\sin A\cos B+\sin B\cos A+\sin A\cos B-\sin B\cos A$ $=2\sin A\cos B$
2ii	$AB+BC=\sin\left(\frac{\pi}{4}+\theta\right)+\sin\left(\frac{\pi}{4}-\theta\right)$ $=2\sin\frac{\pi}{4}\cos\theta$ $\approx2\left(\frac{\sqrt{2}}{2}\right)\left(1-\frac{\theta^2}{2!}+\frac{\theta^4}{4!}\right)$ $=\sqrt{2}\left(1-\frac{\theta^2}{2}+\frac{\theta^4}{24}\right)$ $p=\sqrt{2},\ q=24$

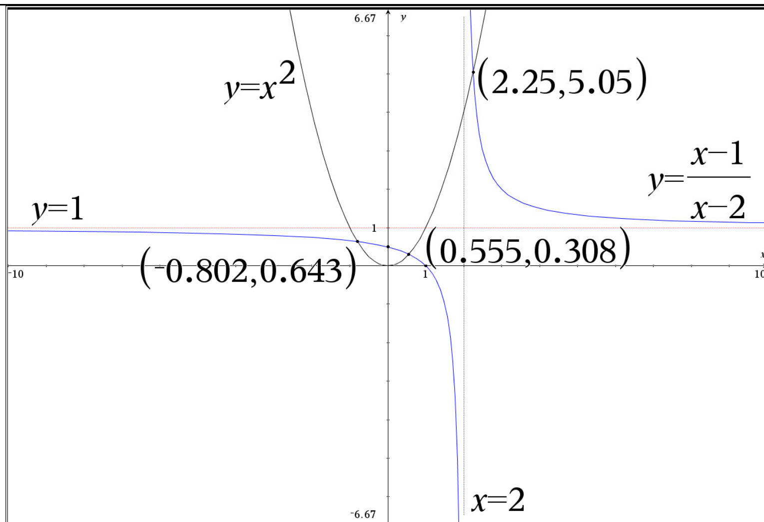
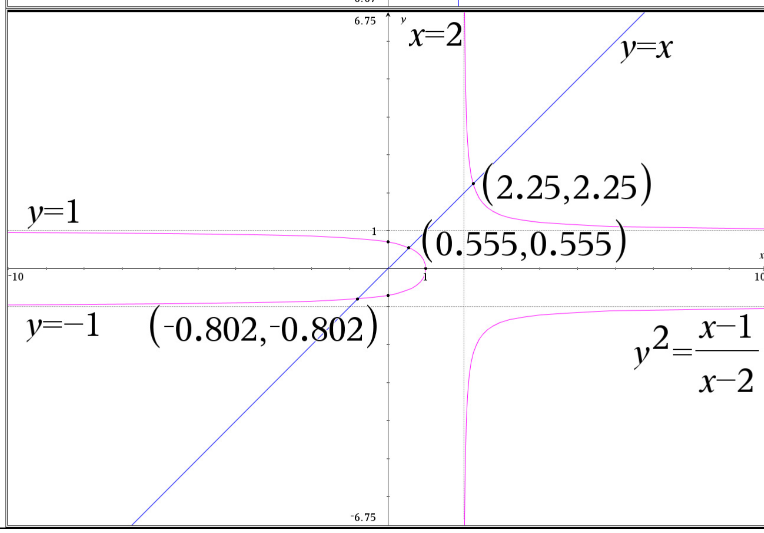
3i	$\frac{\mathrm{d}x}{\mathrm{d}t} = -kx$ $\int \frac{1}{x}\ \mathrm{d}x = -k\int \mathrm{d}t$ $\ln x = -kt + c$ when $t = 0,\ x = x_0$, $c = \ln x_0$ when $x = 0.5x_0$, $\ln 0.5x_0 = -kt + \ln x_0$ $t = -\frac{\ln 0.5}{k}$ $= \frac{\ln 2}{k}$
3ii	$\frac{\ln 2}{k} = 5730$ $k = \frac{\ln 2}{5730}$ when $x = 0.002x_0$, $\ln 0.002x_0 = -\frac{\ln 2}{5730}t + \ln x_0$ $t = -\frac{5730 \ln 0.002}{\ln 2}$ $= 51374$ years
4	$\overrightarrow{OB} = \mathbf{a} + \mathbf{c}$ and $\overrightarrow{AC} = \mathbf{c} - \mathbf{a}$ $\left\|\overrightarrow{OB}\right\|^2 + \left\|\overrightarrow{AC}\right\|^2 = \left\|\mathbf{a}+\mathbf{c}\right\|^2 + \left\|\mathbf{c}-\mathbf{a}\right\|^2$ $= (\mathbf{a}+\mathbf{c})\cdot(\mathbf{a}+\mathbf{c}) + (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{a})$ $= \left\|\mathbf{a}\right\|^2 + 2\mathbf{a}\cdot\mathbf{c} + \left\|\mathbf{c}\right\|^2 + \left\|\mathbf{a}\right\|^2 - 2\mathbf{a}\cdot\mathbf{c} + \left\|\mathbf{c}\right\|^2$ $= \left\|\overrightarrow{OA}\right\|^2 + \left\|\overrightarrow{AB}\right\|^2 + \left\|\overrightarrow{BC}\right\|^2 + \left\|\overrightarrow{CO}\right\|^2$ Alternative solution: $\left\|\overrightarrow{OA}\right\|^2 + \left\|\overrightarrow{AB}\right\|^2 + \left\|\overrightarrow{BC}\right\|^2 + \left\|\overrightarrow{CO}\right\|^2 = \left\|\mathbf{a}\right\|^2 + \left\|\mathbf{c}\right\|^2 + \left\|\mathbf{a}\right\|^2 + \left\|\mathbf{c}\right\|^2$ $= \left\|\mathbf{a}\right\|^2 + 2\mathbf{a}\cdot\mathbf{c} + \left\|\mathbf{c}\right\|^2 + \left\|\mathbf{a}\right\|^2 - 2\mathbf{a}\cdot\mathbf{c} + \left\|\mathbf{c}\right\|^2$ $= (\mathbf{a}+\mathbf{c})\cdot(\mathbf{a}+\mathbf{c}) + (\mathbf{c}-\mathbf{a})\cdot(\mathbf{c}-\mathbf{a})$ $= \left\|\mathbf{a}+\mathbf{c}\right\|^2 + \left\|\mathbf{c}-\mathbf{a}\right\|^2$ $= \left\|\overrightarrow{OB}\right\|^2 + \left\|\overrightarrow{AC}\right\|^2$
4	$\left\|\overrightarrow{OA}\right\|^2 + \left\|\overrightarrow{AB}\right\|^2 + \left\|\overrightarrow{BC}\right\|^2 + \left\|\overrightarrow{CO}\right\|^2 = \left\|\overrightarrow{OB}\right\|^2 + \left\|\overrightarrow{AC}\right\|^2$ $\left\|\overrightarrow{OA}\right\|^2 + \left\|\overrightarrow{OC}\right\|^2 = \left\|\overrightarrow{OB}\right\|^2$ $\left\|\mathbf{a}\right\|^2 + \left\|\mathbf{c}\right\|^2 = \left\|\mathbf{a}+\mathbf{c}\right\|^2$ $= \left\|\mathbf{a}\right\|^2 + 2\mathbf{a}\cdot\mathbf{c} + \left\|\mathbf{c}\right\|^2$ $\mathbf{a}\cdot\mathbf{c} = 0$ $\mathbf{a}$ is perpendicular to $\mathbf{c}$.

5ai	Since T_n is an AP, $T_n = a+(n-1)d$. $\frac{e^{T_{n+1}}}{e^{T_n}} = e^d$ which is still a constant, $\therefore$ the terms of the sequence with nth term e^{T_n} follows a GP.
5aii	Since X_n is a GP, $X_n = ar^{n-1}$. $\ln X_{n+1} - \ln X_n = \ln ar^n - \ln ar^{n-1}$ $= \ln r,$ which is a constant $\therefore$ the terms of the sequence with nth term $\ln X_n$ follows an AP.
5b	$\frac{a+2d}{a} = \frac{a+3d}{a+2d}$ $a^2+4ad+4d^2 = a^2+3ad$ $d(a+4d)=0$ $d=0$ (NA) or $a=-4d$ $r = \frac{-4d+2d}{-4d}$ $= \frac{1}{2}$ Since $\lvert r \rvert < 1,$ GP is convergent.

6i	$\frac{d^2 y}{dx^2}=ae^{2x}+be^{-3x}$ $\frac{dy}{dx}=\frac{a}{2}e^{2x}-\frac{b}{3}e^{-3x}+c$ $y=\frac{a}{4}e^{2x}+\frac{b}{9}e^{-3x}+cx+d$
6ii	From Maclaurin series, when $x=0,\ y=2,\ \frac{dy}{dx}=-1,\ \frac{d^2 y}{dx^2}=13$: $\frac{a}{4}+\frac{b}{9}+0c+d=2$ $\frac{a}{2}-\frac{b}{3}+c=-1$ $a+b+0c+0d=13$ Alternative solution: $y=\frac{a}{4}e^{2x}+\frac{b}{9}e^{-3x}+cx+d$ $\approx\frac{a}{4}\left(1+2x+\frac{4x^2}{2}\right)+\frac{b}{9}\left(1-3x+\frac{9x^2}{2}\right)+cx+d$ $2-x+\frac{13}{2}x^2\approx\left(\frac{a}{4}+\frac{b}{9}+d\right)+\left(\frac{a}{2}-\frac{b}{3}+c\right)x+\left(\frac{a}{2}+\frac{b}{2}\right)x^2$ $\frac{a}{4}+\frac{b}{9}+d=2$ $\frac{a}{2}-\frac{b}{3}+c=-1$ $\frac{a}{2}+\frac{b}{2}=\frac{13}{2}$ Curve passes through $\left(\ln 2,\frac{33}{8}\right)$: $a+\frac{b}{72}+c\ln 2+d=\frac{33}{8}$ Using GC, $a=4,\ b=9,\ c=d=0.$ $\therefore y=e^{2x}+e^{-3x}$

7a	$\int \tan^{-1} x \ \mathrm{d}x = x\tan^{-1} x - \frac{1}{2}\int \frac{2x}{x^2+1} \ \mathrm{d}x + c'$ $= x\tan^{-1} x - \frac{1}{2}\ln\left(x^2+1\right) + c$
7b	$\int \frac{2x}{x^2+2x+1} \ \mathrm{d}x = \int \frac{2x+2}{x^2+2x+1} - \frac{2}{\left(x+1\right)^2} \ \mathrm{d}x$ $= \ln\left(x^2+2x+1\right) + \frac{2}{x+1} + c$ Alternative solution: $\int \frac{2x}{x^2+2x+1} \ \mathrm{d}x = \int \frac{2}{x+1} - \frac{2}{\left(x+1\right)^2} \ \mathrm{d}x$ $= 2\ln\left(x+1\right) + \frac{2}{x+1} + c$
7c	$\int_1^2 \frac{1}{x\sqrt{x^2-1}} \ \mathrm{d}x = \int_1^{\frac{1}{2}} \frac{1}{\frac{1}{u}\sqrt{\frac{1}{u^2}-1}}\left(-\frac{1}{u^2}\right) \ \mathrm{d}u$ $= \int_{\frac{1}{2}}^{1} \frac{1}{\sqrt{1-u^2}} \ \mathrm{d}u$ $= \left[\sin^{-1} u\right]_{\frac{1}{2}}^{1}$ $= \frac{\pi}{3}$ Alternative solution: $\int_1^2 \frac{1}{x\sqrt{x^2-1}} \ \mathrm{d}x = \int_1^{\frac{1}{2}} \frac{1}{\frac{1}{u}\sqrt{\frac{1}{u^2}-1}}\left(-\frac{1}{u^2}\right) \ \mathrm{d}u$ $= \int_1^{\frac{1}{2}} -\frac{1}{\sqrt{1-u^2}} \ \mathrm{d}u$ $= \left[\cos^{-1} u\right]_1^{\frac{1}{2}}$ $= \frac{\pi}{3}$

8i	
8ii	Since the figure is symmetrical, $Area = 4\int_{\frac{\pi}{2}}^{0} (\sin 2t)(-\sin t)\ \mathrm{d}t$ $= 4\int_{0}^{\frac{\pi}{2}} \sin t \sin 2t\ \mathrm{d}t$ $= 8\int_{0}^{\frac{\pi}{2}} \sin^2 t \cos t\ \mathrm{d}t$ $Area = 8\left[\frac{\sin^3 t}{3}\right]_0^{\frac{\pi}{2}}$ $= \frac{8}{3}$
8iii	$y = \sin 2t$ $= 2\sin t \cos t$ $= \pm 2x\sqrt{1-x^2}$ $Volume = 2\pi\int_0^1 \left(2x\sqrt{1-x^2}\right)^2\ \mathrm{d}x$ $= 3.351$ *Alternatively,* $Volume = 2\pi\int_{\frac{\pi}{2}}^{0} (\sin 2t)^2 (-\sin t)\ \mathrm{d}t$ $= 3.351$

9i	 
9ii	$x^3-2x^2-x+1=0$ $x^2(x-2)=x-1$ $x^2=\frac{x-1}{x-2}=y$ for C_1 Alternatively, $x^2=\frac{x-1}{x-2}=y$ for C_1 $x^2(x-2)=x-1$ $x^3-2x^2-x+1=0$ The solutions to the simultaneous equations $y=x^2$ and $y=\frac{x-1}{x-2}$ give the x coordinates of points of intersection between C_1 and $y=x^2$. Refer to graph of C_1. $x=-0.802,\ 0.555,\ 2.25$
9iii	$x^2=\frac{x-1}{x-2}=y$ for C_1 $x=\pm\sqrt{\frac{x-1}{x-2}}=y$ for C_2 $\therefore x=-0.802,\ 0.555,\ 2.25$

10i	$(1-i)^n=\left[\sqrt{2}\left(\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right)\right]^n$ $=2^{\frac{n}{2}}\left(\cos\left(-\frac{n\pi}{4}\right)+i\sin\left(-\frac{n\pi}{4}\right)\right)$ $=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$ *Alternatively,* $(1-i)^n=\left[\sqrt{2}e^{-i\frac{\pi}{4}}\right]^n$ $=\sqrt{2}e^{i\frac{n\pi}{4}}$ $=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$ *Alternatively,* $\lvert 1-i\rvert^n=\sqrt{2}^n$ $=2^{\frac{n}{2}}$ $\arg(1-i)^n=n\arg(1-i)$ $=-n\frac{\pi}{4}$ $\therefore (1-i)^n=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$
10ii	$(1-i)^n=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)$ $\sin\frac{n\pi}{4}=0 \qquad \cos\frac{n\pi}{4}<0$ $\frac{n\pi}{4}=k\pi \qquad \frac{\pi}{2}<\frac{n\pi}{4}<\frac{3\pi}{2}$ $n=4k,\ k\in\mathbb{Z}^+ \qquad 2<n<6$ $\therefore n=4$ *Otherwise…* $(1-i)^n=2^{\frac{n}{2}}\left(\cos\frac{n\pi}{4}-i\sin\frac{n\pi}{4}\right)=2^{\frac{n}{2}}e^{-i\frac{n\pi}{4}}$ $-\frac{n\pi}{4}=(2k+1)\pi$ $n=-4(2k+1),\ k\in\mathbb{Z}^-$ $\therefore n=4$
10iii	Since the coefficients of the equation are real, complex roots will occur in conjugate pairs by Conjugate Roots Theorem. Furthermore, the order of the equation is 4, we would expect 4 roots ie. 0 pair of complex root with 4 real roots or 1 pair of complex conjugate roots with 2 real roots or 2 pairs of complex conjugate roots and no real root.
10iv	$z^4+4=0$ $z^4=-4=4e^{i(-\pi+2k\pi)},\ k=0,\pm1,-2$ $z=\sqrt{2}e^{i\left(\frac{\pi}{4}+\frac{k\pi}{2}\right)},\ k=0,\pm1,-2$

	when $k=0,\ z=\sqrt{2}e^{i\left(\frac{\pi}{4}\right)}=1+i$ when $k=-1,\ z=\sqrt{2}e^{i\left(-\frac{\pi}{4}\right)}=1-i$ when $k=1,\ z=\sqrt{2}e^{i\left(\frac{3\pi}{4}\right)}=-1+i$ when $k=-2,\ z=\sqrt{2}e^{i\left(-\frac{3\pi}{4}\right)}=-1-i$ *Alternatively:* $z^4+4=0$ $\left(z^2-2i\right)\left(z^2+2i\right)=0$ $z=\dfrac{\pm\sqrt{-4(-2i)}}{2}=\pm(1+i)$ $z=\dfrac{\pm\sqrt{-4(2i)}}{2}=\pm(1-i)$ *Alternatively:* Let $z=x+iy$. $(x+iy)^4+4=0$ $x^4+4ix^3y-6x^2y^2-4ixy^3+y^4+4=0$ $x^4-6x^2y^2+y^4+4=0$ and $4x^3y-4xy^3=0$ $4xy\left(x^2-y^2\right)=0$ $x=\pm y$ when $x=y$, when $x=-y$, $y^4=1$ $y^4=1$ $y=\pm1$ $y=\pm1$ $x=\pm1$ $x=\mp1$ $\therefore z=1+i,\ -1-i,\ 1-i,\ -1+i$

11i	$\frac{1}{x_{r-1}x_r}-\frac{1}{x_r x_{r+1}}=\frac{x_{r+1}-x_{r-1}}{x_{r-1}x_r x_{r+1}}$ $=\frac{2x_r}{x_{r-1}x_r x_{r+1}}$ $=\frac{2}{x_{r-1}x_{r+1}}$
11ii	$\sum_{r=2}^{n}\frac{1}{x_{r-1}x_{r+1}}=\frac{1}{2}\sum_{r=2}^{n}\left(\frac{1}{x_{r-1}x_r}-\frac{1}{x_r x_{r+1}}\right)$ $=\frac{1}{2}\left(\frac{1}{x_1x_2}-\frac{1}{x_2x_3}\right.$ $+\frac{1}{x_2x_3}-\frac{1}{x_3x_4}$ $+ \ldots$ $\left.+\frac{1}{x_{n-1}x_n}-\frac{1}{x_n x_{n+1}}\right)$ $=\frac{1}{2}\left(1-\frac{1}{x_n x_{n+1}}\right)$
11iii	Let P_n be the statement $\sum_{r=2}^{n}\frac{1}{x_{r-1}x_{r+1}}=\frac{1}{2}\left(1-\frac{1}{x_n x_{n+1}}\right)$ for $n\in\mathbb{Z}^+,\ n\geq 2$. when $n = 2$, $\text{LHS}=\sum_{r=2}^{2}\frac{1}{x_{r-1}x_{r+1}}=\frac{1}{x_1x_3}=\frac{1}{1\cdot 3}=\frac{1}{3}$ $\text{RHS}=\frac{1}{2}\left(1-\frac{1}{x_2x_3}\right)=\frac{1}{2}\left(1-\frac{1}{1\cdot 3}\right)=\frac{1}{3}=\text{LHS}$ $\therefore P_2$ is true. Assume P_k true for some $k\in\mathbb{Z}^+,\ k\geq 2$ ie $\sum_{r=2}^{k}\frac{1}{x_{r-1}x_{r+1}}=\frac{1}{2}\left(1-\frac{1}{x_k x_{k+1}}\right)$ To prove P_{k+1} is true $\sum_{r=2}^{k+1}\frac{1}{x_{r-1}x_{r+1}}=\frac{1}{2}\left(1-\frac{1}{x_{k+1}x_{k+2}}\right)$ when $n = k + 1$, $LHS=\frac{1}{2}\left(1-\frac{1}{x_k x_{k+1}}\right)+\frac{1}{x_k x_{k+2}}$ $=\frac{1}{2}\left(1-\frac{1}{x_k x_{k+1}}+\frac{1}{x_k x_{k+1}}-\frac{1}{x_{k+1}x_{k+2}}\right)$ $=\frac{1}{2}\left(1-\frac{1}{x_{k+1}x_{k+2}}\right)=\text{RHS}$ P_k true $\Rightarrow P_{k+1}$ true. Since P_2 true and P_k true $\Rightarrow P_{k+1}$ true, P_n true for all $n\in\mathbb{Z}^+,\ n\geq 2$.
11iv	Since $x_n\to\infty$ as $n\to\infty$, $\frac{1}{x_n x_{n+1}}\to 0$ and $\sum_{r=2}^{n}\frac{1}{x_{r-1}x_{r+1}}\to\frac{1}{2}$ as $n\to\infty$.

12i	$\overrightarrow{AB}=\begin{pmatrix}-9+3\lambda\\15-7\lambda\\-4+4\lambda\end{pmatrix}$ $\left\|\overline{\text{AB}}\right\|^2=(-9+3\lambda)^2+(15-7\lambda)^2+(-4+4\lambda)^2=10^2$ $\lambda^2-4\lambda+3=0$ $\lambda=3$ or $\lambda=1$ $\overrightarrow{OB}=\begin{pmatrix}2\\-6\\7\end{pmatrix}$ $\overrightarrow{OC}=\begin{pmatrix}-4\\8\\-1\end{pmatrix}$
12ii	$\overrightarrow{OM}=\frac{1}{2}\left[\begin{pmatrix}2\\-6\\7\end{pmatrix}+\begin{pmatrix}-4\\8\\-1\end{pmatrix}\right]=\begin{pmatrix}-1\\1\\3\end{pmatrix}$ $\mathbf{n}=\overrightarrow{AM}=\begin{pmatrix}-1\\1\\3\end{pmatrix}-\begin{pmatrix}2\\0\\-1\end{pmatrix}=\begin{pmatrix}-3\\1\\4\end{pmatrix}$ $\mathbf{r}\cdot\begin{pmatrix}-3\\1\\4\end{pmatrix}=\begin{pmatrix}-7\\15\\-5\end{pmatrix}\cdot\begin{pmatrix}-3\\1\\4\end{pmatrix}$ $-3x+y+4z=21+15-20$ $=16$
12iii	Since $\begin{pmatrix}2\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\2\\-3\end{pmatrix}=5$ and $\begin{pmatrix}2\\0\\-1\end{pmatrix}\cdot\begin{pmatrix}1\\-2\\1\end{pmatrix}=1$, A lies in both π_1 and π_2.
12iv	$-3x+y+4z=16$ $x+2y-3z=5$ $x-2y+z=1$ Using GC, $x=15,\ y=13,\ z=12$ $\overrightarrow{OD}=\begin{pmatrix}15\\13\\12\end{pmatrix}$
12v	perpendicular height $=\left\|\overrightarrow{AM}\right\|$ $=\sqrt{9+1+16}$ $=\sqrt{26}$

	$\overrightarrow{BD}=\begin{pmatrix}13\\19\\5\end{pmatrix},\ \overrightarrow{CD}=\begin{pmatrix}19\\5\\13\end{pmatrix}$ $\text{area of base}=\frac{1}{2}\left\lvert\overrightarrow{BD}\times\overrightarrow{CD}\right\rvert$ $=\frac{1}{2}\left\lvert\begin{pmatrix}222\\-74\\-296\end{pmatrix}\right\rvert$ $=\sqrt{35594}$ $\text{volume}=\frac{1}{3}\times\sqrt{26}\times\sqrt{35594}$ $=\frac{962}{3}\text{ units}^3$