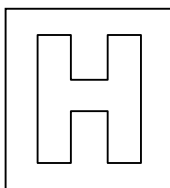


Candidate Name: _____

Class	Adm No



2014 Preliminary Examination II

Pre-university 3

MATHEMATICS

9740/02

Paper 2

24 September 2014

3 hours

Additional Materials: Answer Paper
 List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER and fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

This question paper consists of 7 printed pages.

[Turn over

Section A: Pure Mathematics [40 marks]

- 1 (i) Without the use of a calculator, solve the inequality

$$\frac{-x^2 + 4x + 2}{x + 4} \geq 1. \quad [4]$$

- (ii) Hence, solve the inequality

$$\frac{-2\sin^2 \theta + 4\sin \theta + 1}{\sin \theta + 2} \geq 1$$

where $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$. [2]

- 2 Benjamin wants to make an open fish tank in the shape of a cuboid with a square base. The base of the fish tank is to be made from an opaque material which costs \$5 per square centimeter. The sides of the fish tank are to be made from a transparent material which costs \$2 per square centimeter. All the materials used are of negligible thickness.

If the fish tank has a capacity of $10\,000 \text{ cm}^3$, find the minimum cost of the fish tank. [6]

- 3 The complex number z satisfies the relations

$$0 \leq \arg(z+1) \leq \frac{\pi}{4} \quad \text{and} \quad |z-3| \leq |z-3-6i|.$$

- (i) Draw an Argand diagram to illustrate the locus of z . [3]

- (ii) Find the least value of $|z+2-7i|$ in exact form. [2]

- (iii) Find the range of values of $\arg(z+2-3i)$. [2]

- 4 A curve C has parametric equations

$$x = \ln t, \quad y = 2 - \frac{1}{2t}.$$

- (i) Show that the equation of the tangent to the curve C at the point with parameter p is

$$y = \frac{x}{2p} - \frac{\ln p}{2p} - \frac{1}{2p} + 2. \quad [3]$$

- (ii) Let A be the point on the curve C with parameter 1. The tangent and normal at A intersect the x -axis at the points T and N respectively. Find the coordinates of the points T and N and the area of the triangle ANT . [5]

- 5 The functions f and g are defined by

$$f(x) = \frac{1}{2-x} + 3, \quad x > 2, \quad x \in \mathbb{R},$$

$$g(x) = f(x), \quad x > a, \quad x \in \mathbb{R}$$

where a is a real constant such that $a > 2$.

- (a) (i) Sketch the graph of $y = f(x)$ stating clearly the equation of any asymptotes and the coordinates of any points of intersection with the axes. [3]
- (ii) Define f^{-1} in a similar form. [2]
- (iii) State the range of the function g in terms of a . Hence, determine the least value of a such that the composite function fg exists. [2]
- (iv) Using the value of a found in part (iii), define the composite function fg in a similar form and state the range of fg . [3]
- (b) Express $f(x)$ in ascending powers of x up to and including the term in x^3 . [3]

Section B: Statistics [60 marks]

- 6 A company has 1000 employees. 400 employees work during the morning shift, 350 employees work during the afternoon shift and the rest work during the night shift. The director of the company wants to interview a random sample of 100 employees to find out their level of job satisfaction.
- (i) Explain what is meant in this context by the term 'a random sample'. [2]
- (ii) Describe how simple random sampling can be used to select the employees in the required sample. [2]
- (iii) State a more suitable sampling method. Give an advantage of the sampling method that you have stated over simple random sampling. [2]

- 7 A school lunch time concert is held from 1 pm to 2 pm. The number of students who stop to watch the concert in a 1-minute interval follows a Poisson distribution with mean 6. The number of students who leave the concert venue in a 1-minute interval follows a Poisson distribution with mean 3. Assume that the number of students stopping to watch and the number of students leaving are independent of each other.
- (i) Find the probability that at least 15 students stop to watch the concert in a 3-minute interval. [2]
 - (ii) The probability that at least one student leaves the concert venue in a t -minute interval is 0.6. Find t to 5 decimal places. [2]
 - (iii) At 1.15 pm, there are 20 students watching the concert. Using a suitable approximation, find the probability that there are at least 30 but less than 40 students watching the concert by 1.20 pm. [4]
- 8 A machine is used to pack rice into packets of 10 kg. After a few years, the owner started receiving complaints that the mean weight of rice in a packet is not 10 kg. The owner starts to suspect that the machine is faulty due to wear and tear.

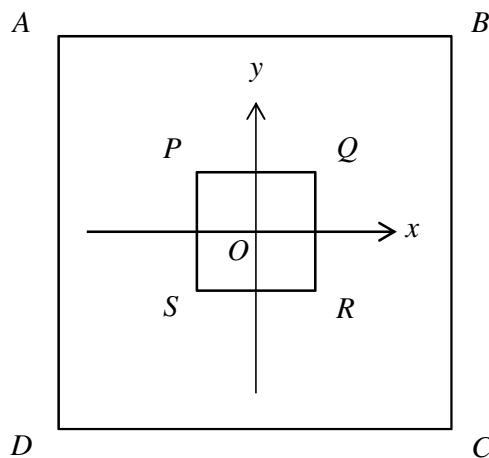
He then proceeds to weigh the amount of rice in 13 packets. The results are summarised below:

$$\sum (x-10) = 2.6 \quad \sum (x-10)^2 = 0.9$$

where x denotes the weight of rice in one packet in kg.

- (i) Calculate the unbiased estimates for the mean and variance of the weight of rice in one packet. [2]
- (ii) Test at the 5% significance level if the owner's suspicion is justifiable, stating a necessary assumption to conduct the test. [5]
- (iii) The same test in part (ii) was conducted but at a different significance level of $\alpha\%$. State the least value of α so that the null hypothesis is rejected. [1]

- 9 A group of friends decided to use a square piece of paper with a side length of 10 cm as a dart board. They labelled the center of the paper as the origin and drew a pair of x -axis and y -axis through the center.



The points A , B , C and D represent the four corners of the square dart board. The square $PQRS$ denotes the “bull’s-eye region”. $PQRS$ is centered at the origin and has a side length of 2 cm.

A dart is thrown and the coordinates of the landing point is (X, Y) , where X and Y are independent random variables such that $X \sim N(0, 4)$ and $Y \sim N(0, 9)$.

- (i) Show that the probability that a dart lands in the bull’s-eye region is 0.099988, correct to 5 significant figures. [1]
- (ii) Find the probability that a dart lands on the dart board but not in the bull’s-eye region. [2]
- (iii) Given that a dart lands on the dart board, find the probability that it did not land in the bull’s-eye region. [2]

A dart that lands in the bull’s-eye region scores 10 points.

A dart that lands on the dart board but not in the bull’s-eye region scores 5 points.

A dart that lands outside the dart board scores no points.

- (iv) Two darts are thrown. Find the probability that a total of 10 points is scored. [3]

- 10** Sarah wants to investigate the growth rate of a particular plant species. She measures the height of the plant, h cm, at the end of each week. Let t be the number of weeks from the start of the experiment.

t (weeks)	1	2	3	4	5	6	7
h (cm)	5.4	14.9	18.8	20.7	21.6	22.4	22.7

- (i) Draw a scatter diagram for these values, labelling the axes. [2]
- (ii) Sarah is considering between the two models below for modelling the growth of the plant species.
- (A) $h = at + b$
- (B) $h = \frac{a}{t} + b$

Calculate the product moment correlation coefficient for both model (A) and model (B). Determine which model is more appropriate for modelling these values. [2]

It is desired to estimate the value of t when $h = 22$.

- (iii) Using model (B) in part (ii), find the equation of a suitable regression line and use this equation to find the required estimate. [2]
- (iv) Sarah's friend used model (A) in part (ii) and wrote the equation of the regression line of h on t in the form $h = at + b$. Predict what will happen to the values of a and b if all the values of h are increased by k cm. [2]
- 11** A machine is used to generate codes consisting of 4 characters. A character is either one of the 26 letters of the alphabet A – Z or one of the 10 digits 0 – 9. Each of the 36 characters are equally likely to be generated and each character can be generated more than once.

Find the probability that a randomly generated code has

- (i) no digits, [1]
- (ii) the first two characters consisting of digits and the first digit is higher than the second digit, [2]
- (iii) three different letters and one digit, [3]
- (iv) exactly two letters and two digits, such that the letters are the same or the digits are the same, but not both. [3]

- 12** A factory manufactures torchlights and two types of batteries. The two types of batteries are named 'Standard' and 'Long Lasting'. The torchlight only requires one battery to work.

When the batteries are used with a working torchlight, the lifetime of a Standard battery follows a normal distribution with mean 300 hours and standard deviation 30 hours and the lifetime of a Long Lasting battery follows a normal distribution with mean 500 hours and standard deviation 50 hours. The lifetimes of a Standard battery and a Long Lasting battery are independent of each other.

For parts (i) and (ii), assume that all batteries are used with a working torchlight.

- (i) Find the probability that a Standard battery lasts longer than 310 hours. [1]
- (ii) Find the probability that the total lifetime of 10 Standard batteries is less than 5 times the lifetime of a Long Lasting battery. [3]

97% of the torchlights manufactured are in working condition. The torchlights are packed in boxes with 10 torchlights in each box.

- (iii) Find the probability that in a randomly chosen box, at least 9 torchlights are in working condition. [1]
- (iv) A shop owner bought 15 boxes of torchlights. Find the probability that more than 12 but fewer than 15 boxes have at least 9 torchlights that are in working condition in each box. [2]
- (v) Using a suitable approximation, find the probability that in 10 boxes of torchlights, less than 96 of all the torchlights are in working condition. [3]
- (vi) Bryan bought one torchlight, one Standard battery and one Long Lasting battery. Find the probability that he will be able to use the torchlight for at least 750 hours. [3]

End of Paper