Section A: Pure Mathematics [40 marks]

1. **(i)** Find the expansion of $\dfrac{1}{\sqrt{k+2x}}$, where $k \in \mathbb{R}$, $k > 1$, in ascending powers of x, up to and including the term in x^2. [3]

(ii) State the range of values of x for which this expansion is valid, giving your answer in terms of k. [1]

(iii) By letting $k = 2$ in part **(i)** and choosing a suitable value of x, estimate the value of $\sqrt{5}$ in the form $\dfrac{m}{n}$ where $m, n \in \mathbb{Z}^+$. [2]

2. The planes p_1 and p_2 have Cartesian equations $2x + y - 3z = -5$ and $5x - 7y + 2z = -3$ respectively and meet in a line ℓ.

(i) Find the acute angle between p_1 and p_2 . [2]

(ii) Find a vector equation of ℓ. [1]

The line ℓ_1 passes through the point A with position vector $-2\mathbf{k}$, is parallel to p_1 and is perpendicular to ℓ.

(iii) Explain briefly why ℓ and ℓ_1 are skew lines. [2]

(iv) Find a vector equation of ℓ_1. [2]

The plane p_3 contains ℓ_1 and is parallel to p_1.

(v) Find the perpendicular distance between p_1 and p_3.

Hence state the perpendicular distance between ℓ and ℓ_1. [3]

3. **(a)** **(i)** Find the fifth roots of –32, expressing the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [2]

(ii) The roots representing z_1 and z_2 are such that $0 < \arg(z_1) < \arg(z_2) < \pi$.

State the complex number w in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $z_2 = wz_1$. [1]

(b) The complex number z satisfies $|z - 3 - 3\mathrm{i}| \ge |z - 1 - \mathrm{i}|$ and $\dfrac{\pi}{6} < \arg(z) \le \dfrac{\pi}{3}$.

(i) On an Argand diagram, sketch the region in which the point representing z can lie. [3]

(ii) Find the area of the region in part **(b)(i)**. [3]

(iii) Find the range of values of $\arg(z - 5 + \mathrm{i})$. [2]

4. The function f is defined by $\mathrm{f} : x \to x|x-3|, \quad x \in \mathbb{R},\ x < m$.

(i) Show that f^{-1} does not exist if $m = 4$. [1]

(ii) State the largest possible value of m for which f^{-1} exists. [1]

Use the value of m obtained in part **(ii)** for the following questions.

(iii) Find f^{-1}, stating its domain. [4]

(iv) The composite function fg is defined by $\mathrm{fg} : x \to \dfrac{9}{4} - x^2, \quad x \in \mathbb{R}, x < 0$.

Find $\mathrm{g}(x)$. [2]

(v) The function h is defined by $\mathrm{h} : x \to \dfrac{3}{x-a}, \quad x \in \mathbb{R}, x < a$.

(a) The graph of $y = \mathrm{k}(x)$ is transformed by a stretch with scale factor 2 parallel to the x-axis, followed by a translation of 10 units in the negative y-direction. The equation of the resulting curve is $y = \mathrm{h}(x)$.

Find $\mathrm{k}(x)$. [2]

(b) Find the smallest value of a such that the composite function hf exists. For this value of a found, determine the range of hf. [3]

Section B: Statistics [60 marks]

5. A group of 5 boys and 5 girls are to be seated in a row of 10 adjacent seats to watch a performance. Find the number of possible seating arrangements if not all the girls are seated together. [2]

After the performance, the group books a round table with 10 identical seats at a restaurant for dinner. Given that 2 particular girls are seated directly opposite each other, find the probability that exactly 4 boys are seated together at the round table during dinner. [3]

6. A survey is conducted on 50 male students and 50 female students in a school to find out which science subject(s) they are studying. The result is recorded in the table shown below.

	Male	**Female**
Study Physics	24	8
Study Biology	33	37
Neither studying Physics nor Biology	8	11

One of the 100 students is selected at random.

(i) Show that the probability that the student studies Physics only is $\frac{11}{100}$. [2]

(ii) Find the probability that the student studies both Physics and Biology given that the student is a male. [2]

(iii) Let X be the event that the student selected is a female, and Y be the event that the student selected studies Physics. Determine whether the events X and Y are independent, justifying your answer. [2]

7. The speed of a randomly chosen passenger car travelling from point A to point B on an expressway has a normal distribution with mean 85 km/h and standard deviation 20 km/h. It is assumed that a passenger car travels at a constant speed from point A to point B.

(i) For passenger cars travelling from point A to point B, find the probability that the total speed of 3 randomly chosen cars differs from twice the speed of a randomly chosen car by at most 50 km/h. [3]

(ii) The speeds of 80 randomly chosen passenger cars which travelled from point A to point B are recorded. Find the probability that the total distance travelled in 5 minutes is at most 550 km. State an assumption you had made in your calculation.
[4]

8. The height, H cm, of any randomly chosen kindergarten pupil in Singapore follows a normal distribution with mean μ and variance σ^2.

(i) It is given that $\mathrm{P}(H < 115) = 0.95$ and $\mathrm{P}(H > 80) = 0.75$. Find the values of μ and σ. [3]

A university student doing Health Sciences wishes to take a sample of 60 pupils to investigate the heights of kindergarten pupils in Singapore for her project.

(ii) Using a suitable approximation, find the probability that there are more than 57 pupils who are shorter than 1.15 m. [3]

(iii) State and describe a suitable sampling method that the university student can use to obtain her sample. [2]

9. The radiation intensity I from a radioactive source for different time t, in appropriate units, is recorded. The results are shown in the table.

t	1	2	3	4	5	6
I	20	12.6	8.8	7.5	7.3	7

(i) Draw a scatter diagram for the data. [2]

(ii) Without calculating the product moment correlation coefficients, explain why model C below is more appropriate for modelling the data, and why the other 2 models are not suitable:

(A) $I = a + bt$,

(B) $I = a + bt^2$,

(C) $I = ae^{bt}$,

where a is positive and b is negative in all three models. [2]

(iii) Calculate the least squares estimates of a and b for model C. Give an interpretation, in context, of the value of a obtained. [3]

(iv) Predict the value of t when $I = 8.0$. Comment on the reliability of your prediction.

[2]

10. A company manufactures coffee capsules. The mass (in grams) of coffee in one capsule is denoted by X. The company claims that the mean mass of coffee in a capsule is 5 grams. A consumer who regularly buys coffee capsules wants to know if the company has overstated its claim. He measures the mass of coffee in 9 random capsules, which are summarised by

$$\sum(x-4)=8.0, \quad \sum(x-4)^2=7.38.$$

(i) Show that the unbiased estimate of the variance of X is 0.0336, correct to 3 significance figures. [2]

(ii) Stating a necessary assumption, carry out a suitable test at the 5% significance level. [4]

The company knows that X is normally distributed with standard deviation of 0.25. The company directors want to carry out a test to determine whether their claim is incorrect. A random sample of 100 capsules is used to carry out the test. The mean mass of coffee in this sample of 100 capsules is denoted as $\overline{x}$.

(iii) State the appropriate hypotheses for the test. [1]

(iv) Use an algebraic method to calculate the range of values of $\overline{x}$ for which the null hypothesis would not be rejected at the 5% significance level. [3]

(v) Given that the range of values of $\overline{x}$ for which the null hypothesis would not be rejected is $4.94<\overline{x}<5.06$, calculate the probability that the null hypothesis is rejected given that it is actually true. [2]

11. A company has 3 cameras for rental at \$200 each per day. The number of customers wanting to rent cameras on a randomly chosen day is assumed to follow a Poisson distribution with mean 2. The company operates every day. It is also assumed that each customer will rent exactly one camera for only a day.

(i) Find the probability that no camera is rented on a randomly chosen day. [1]

(ii) Find the probability that the company cannot meet the customers' demand on a randomly chosen day. [2]

(iii) Find the probability that there are no more than 3 days in a particular week where no cameras are rented. [2]

(iv) Find the probability that, out of 3 days, there are 2 days with no rental and 1 day where the company cannot meet the customers' demand. [2]

(v) Using a suitable approximation, find the probability that the number of customers who want to rent a camera in a week is not more than 8. [3]

(vi) A random sample of 52 weeks is chosen. Find the probability that the average number of days in a week where no cameras are rented is at most 1. [3]