1 Adam bought a total of 50 fruits consisting of apples, oranges and pineapples. The apples, oranges and pineapples cost \$0.80, \$0.60 and \$1.20 each respectively. The total cost of all the fruits he bought is \$40. If the cost of apples is doubled and that of oranges is halved, then the total cost of all the fruits that he bought would be \$53. Find the number of each type of fruit bought by Adam. [3]

2 It is given that x, y, z are the first three terms of a geometric progression. When the three terms are arranged in the order of z, x, y, they form three consecutive terms of an arithmetic progression.

(i) Show that $\left(\frac{z}{y}\right)^2+\left(\frac{z}{y}\right)-2=0$. [4]

(ii) Hence determine if the sum to infinity of the geometric progression exists. [2]

3 **(i)** Without the use of a calculator, solve the inequality $\frac{\left(x^2-2x+4\right)\left(x-3\right)}{\left(x+2\right)}\geq 0$. [5]

(ii) Hence solve the inequality $\frac{\left(x^2-2|x|+4\right)\left(|x|-3\right)}{\left(|x|+2\right)}\geq 0$. [2]

4 **(i)** Solve the equation $z^4=\sqrt{3}-\mathrm{i}$, giving the roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$ where $r>0$ and $-\pi<\theta\leq\pi$. [4]

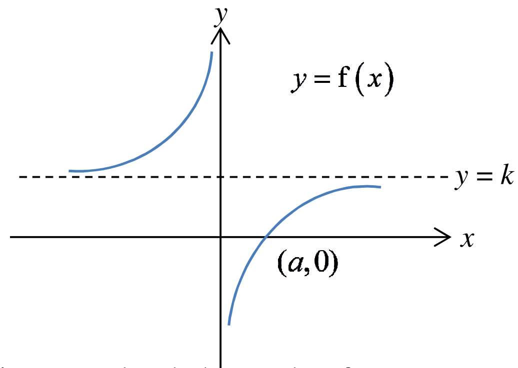
(ii) Show the roots on an Argand diagram and state the cartesian equation of a geometrical shape that the roots lie on. [3]

5 **(i)** Express $\frac{5+x^2}{\left(2+x\right)\left(1-x\right)^2}$ in the form of $\frac{A}{\left(2+x\right)}+\frac{B}{\left(1-x\right)^2}$, where A and B are constants to be found. [3]

(ii) Hence, expand $\frac{5+x^2}{\left(2+x\right)\left(1-x\right)^2}$ as a series of ascending powers of x up to and including the x^2 term. [4]

(iii) State the range of values of x for which the expansion is valid. [1]

6 **(a)** The diagram shows the graph of $y = \mathrm{f}(x)$ with asymptotes $y = k$, $x = 0$ and the graph cuts the x-axis at $(a, 0)$.



On separate diagrams, sketch the graphs of

(i) $y = \dfrac{1}{\mathrm{f}(x)}$ and

(ii) $y = \sqrt{\mathrm{f}(-x)}$,

giving the equations of any asymptotes and the coordinates of any points where the curves cross the x- and y-axes. [4]

(b) The curve $y = \mathrm{g}(x)$ undergoes the transformations A, B and C in succession:

A. A stretch parallel to the x-axis with scale factor 2,
B. A reflection in the y-axis, and
C. A translation of 1 unit in the direction of the y-axis.

Find an expression for $\mathrm{g}(x)$ if the equation of the resulting curve is $y = 1 - \dfrac{1}{x}$. [3]

7 A sequence u_1, u_2, u_3, … is such that $u_1 = \dfrac{3}{4}$ and $u_{n+1} = u_n + \dfrac{3}{4}\left(\dfrac{1}{2}\right)^{2n}$ for all $n \geq 1$.

(i) Write down the values of u_2, u_3 and u_4. [1]

(ii) By considering $1 - u_n$ or otherwise, write down a conjecture for u_n. Use the method of mathematical induction to prove the conjecture. [5]

(iii) Hence find $\displaystyle\sum_{r=2}^{N} \frac{3}{4}\left(\frac{1}{2}\right)^{2r}$ in terms of N. [2]

(iv) Find the smallest value of N such that $\displaystyle\sum_{r=2}^{N} \frac{3}{4}\left(\frac{1}{2}\right)^{2r}$ exceeds $\dfrac{3}{50}$. [2]

8 **(a)** Find the exact value of the constant p such that

$$\int_5^{p+9} \frac{1}{\sqrt{9-x}}\,\mathrm{d}x = \int_0^{\frac{1}{4}} \frac{1}{\sqrt{1-4x^2}}\,\mathrm{d}x. \qquad [4]$$

(b) Use the substitution $x = \cos^2\theta$ to find the exact value of $\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}}\,\mathrm{d}x$. [5]

9 Relative to the origin O, the position vectors of points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are non-zero and non-parallel vectors. The point P on OA is such that $OP : PA = 2 : 3$. The point Q is such that $OPQB$ is a parallelogram.

(i) Find $\overrightarrow{OQ}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. [3]

(ii) Show that the area of the triangle OAQ can be written as $k|\mathbf{a}\times\mathbf{b}|$, where k is a constant to be found. [2]

(iii) State the ratio of the area of triangle OPB to area of triangle OAB. [1]

(iv) Given $\mathbf{a}\times\mathbf{b}$ is a unit vector, $|\mathbf{a}| = 2$ and the angle between $\mathbf{a}$ and $\mathbf{b}$ is 60°, find the exact value of $|\mathbf{b}|$. [3]

10 **(i)** On a single Argand diagram, sketch the locus of points representing the complex number z such that

$$|z-4-2\mathrm{i}| \le 2 \qquad \text{and} \qquad |z-3| \le |z-5|. \qquad [3]$$

(ii) Find the greatest and least possible values of

(a) $|z|$, [4]

(b) $\arg(z)$. [3]

11 The curve C has equation $y = x\cos 2x$, where $0 \le x \le \pi$.

(i) Find the exact x-coordinates of the points where C crosses the x-axis. [3]

(ii) Sketch C, stating the coordinates of any points where the curve crosses the x- and y-axes. [2]

(iii) Find the exact value of $\int_{\frac{\pi}{4}}^{\pi} |x\cos 2x|\,\mathrm{d}x$. [4]

(iv) Find the volume of revolution when the region bounded by the curve C, the x-axis and the line $x = \pi$ is rotated completely about the x-axis. [2]

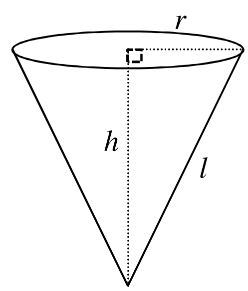
12 [It is given that a cone with radius r, height h and slant height l has curved surface area πrl.]

(a) A drinking cup is manufactured in the shape of a cone. It has a volume of 50π cm^3. Show that

$$A^2 = \frac{22500\pi^2}{r^2} + \pi^2 r^4,$$

where A is the curved surface area of the cone.

Use differentiation to find the height h cm and radius r cm of the cup that will require the least amount of material. [8]



(b) Another drinking cup of the same shape is manufactured. At the instant when the depth of water in the drinking cup is h cm, the volume V cm^3 of the water is given by $V = \dfrac{\pi h^3}{12}$. The cup is filled and it is discovered that there is a leak at the vertex of the cup and the volume of water in the cone is decreasing at the constant rate of $3\,\text{cm}^3\text{s}^{-1}$. Calculate,

(i) the rate at which the depth is decreasing at the instant when the depth is 2 cm, [3]

(ii) the time taken in seconds for the depth to decrease from 6 cm to 3 cm. [2]

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