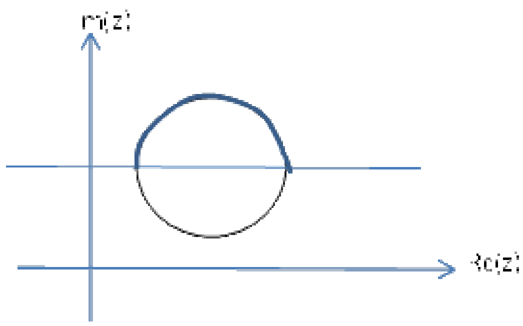
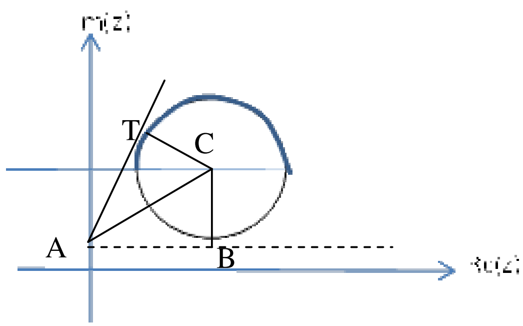
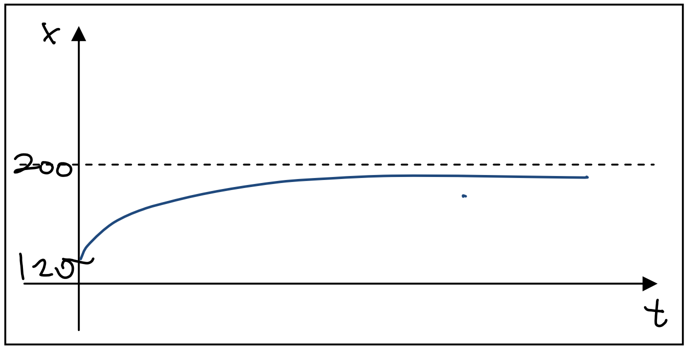
1	
(i)	Least $\lvert z\rvert=\sqrt{4^2+2^2}$ $= 2\sqrt{5}$
(ii)	 Using coordinates A(0,2), C(3,4) and B(3,2), Angle CAB = $\tan^{-1}\left(\dfrac{2}{3}\right)$

	AC = $\sqrt{13}$ Angle TAC = $\sin^{-1}\left(\frac{1}{\sqrt{13}}\right)$ Largest arg (z – 2i) = Angle TAB $= \tan^{-1}\left(\frac{2}{3}\right)+\sin^{-1}\left(\frac{1}{\sqrt{13}}\right)$

2(i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=2x-0.01x^2=0.01x\left(200-x\right)$
	$\int\frac{1}{x\left(200-x\right)}\mathrm{d}x=\int 0.01\mathrm{d}t$
	$\frac{1}{200}\int\frac{1}{x}+\frac{1}{200-x}\mathrm{d}x=\int 0.01\mathrm{d}t$
	$\frac{1}{200}\left[\ln\lvert x\rvert-\ln\lvert 200-x\rvert\right]=0.01t+c$
	$\ln\left\lvert\frac{x}{200-x}\right\rvert=200\left(0.01t+c\right)=2t+C$
	$\frac{x}{200-x}=\pm e^{2t+c}=Be^{2t}$
	$x\left(1+Be^{2t}\right)=200Be^{2t}\Rightarrow x=\frac{200Be^{2t}}{1+Be^{2t}}$
	Divide by Be^{2t} throughout: $x=\frac{200}{\frac{1}{Be^{2t}}+1}=\frac{200}{Ae^{-2t}+1}$ (shown)
(ii)	When t = 0, x = 120, $120=\frac{200}{A+1}\Rightarrow A=\frac{2}{3}$

	When $t = 2$, $x = \dfrac{200}{\frac{2}{3}e^{-2(2)}+1} = 197.587$ There are 197587 prawns after 2 weeks. Correcting to the nearest hundred, we have <u>197600 prawns</u>
(iii)	As $t \to \infty, x \to 200$ And so the <u>population of prawns stabilizes at 200 thousands</u>
(iv)	When $t = 0$, $x = 120$ 
	If the initial value = 300 thousands, the population of the prawns will <u>decrease & stabilizes at</u> <u>200 thousands.</u>

3(i)	*O*, *A* , *B* and *C* are coplanar **<u>OR</u>** *O*, *A* , *B* and *C* lie on the same plane.
(ii)	$\overrightarrow{AC} = \overrightarrow{OB}$ $\mathbf{c} - \mathbf{a} = \mathbf{b}$ $\mathbf{c} = \mathbf{a} + \mathbf{b}$ $(\mathbf{a} \bullet \mathbf{a})\mathbf{a} + (\mathbf{a} \bullet \mathbf{b})\mathbf{b} = \mathbf{a} + \mathbf{b}$ $\Rightarrow \lvert\mathbf{a}\rvert^2 = 1$ and $\mathbf{a} \bullet \mathbf{b} = 1$ $\Rightarrow \lvert\mathbf{a}\rvert = 1$ So **a** is a unit vector.

(iii)	By ratio theorem, $\overrightarrow{ON}=\dfrac{\overrightarrow{OU}+2\overrightarrow{OV}}{3}$ $=\dfrac{1}{3}\left(\dfrac{1}{2}\mathrm{a}+\dfrac{6}{5}\mathrm{b}\right)=\dfrac{1}{6}\mathrm{a}+\dfrac{2}{5}\mathrm{b}$
(iv)	Area of $OUN=\dfrac{1}{2}\mid\overrightarrow{OU}\times\overrightarrow{ON}\mid$ $=\dfrac{1}{2}\left\|\dfrac{1}{2}a\times\left(\dfrac{1}{6}a+\dfrac{2}{5}b\right)\right\|$ $=\dfrac{1}{2}\left\|\dfrac{1}{12}(a\times a)+\dfrac{1}{5}(a\times b)\right\|$ $=\dfrac{1}{10}\|a\times b\|$ $=\dfrac{1}{10}\|\mathrm{a}\|\|\mathrm{b}\|\sin\dfrac{\pi}{3}$ $=\dfrac{1}{10}\left(\dfrac{\sqrt{3}}{2}\right)\|\mathrm{b}\|$ $=\dfrac{\sqrt{3}}{20}\|\mathrm{b}\|$

4(a)	$(4+8x)^{\frac{1}{2}}=2(1+2x)^{\frac{1}{2}}$

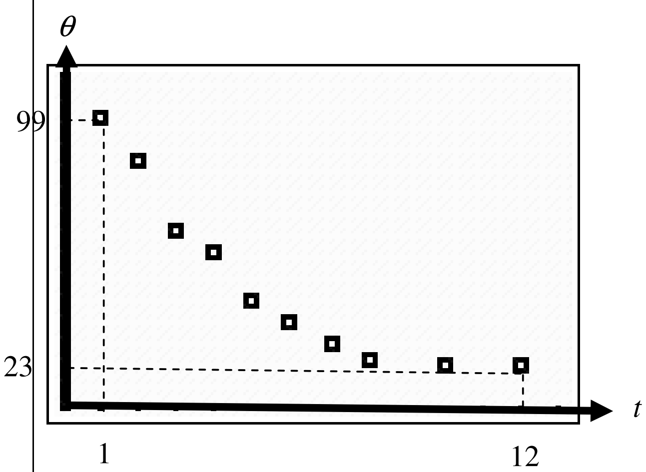
	$= 2(1+\frac{1}{2}(2x)+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2!}(2x)^2$ $+\frac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(2x)^3)+..........$ $= 2(1+x-\frac{1}{2}x^2+\frac{1}{2}x^3+....)$
(b)	$y=\ln(\cos x)$ $\frac{dy}{dx}=\frac{-\sin x}{\cos x}=-\tan x$ $\frac{d^2y}{dx^2}=-\sec^2 x=-1-\tan^2 x$ $\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2+1=0$ (shown) $\frac{d^3y}{dx^3}+2\left(\frac{dy}{dx}\right)\left(\frac{d^2y}{dx^2}\right)=0$ $\frac{d^4y}{dx^4}+2\left(\frac{dy}{dx}\right)\left(\frac{d^3y}{dx^3}\right)+2\left(\frac{d^2y}{dx^2}\right)^2=0$ When $x=0$, $y=0, \frac{dy}{dx}=0, \frac{d^2y}{dx^2}=-1,$ $\frac{d^3y}{dx^3}=0, \frac{d^4y}{dx^4}=-2$ $y=-\frac{1}{2!}x^2-\frac{2}{4!}x^4+.......$

	$y=-\frac{1}{2}x^2-\frac{1}{12}x^4+\ldots\ldots.$
	$\sqrt{4+16x}-\tan x=\sqrt{4+8(2x)}-\frac{dy}{dx}$ $=2(1+2x-\frac{1}{2}(2x)^2+\frac{1}{2}(2x)^3)+\frac{d}{dx}\left(-\frac{1}{2}x^2-\frac{1}{12}x^4\right)+\ldots$ $=2+4x-4x^2+8x^3-x-\frac{1}{3}x^3+\ldots..$ $=2+3x-4x^2+\frac{23}{3}x^3+\ldots..$

Statistics

5(i)	The probability of obtaining a faulty item is a constant. The event of obtaining a faulty item is independent of other event of obtaining a faulty item.
(ii)	Let F be the random variable denoting the number of faulty items assembled out of 15 items. $F \sim \mathrm{B}(15, 0.04)$ $\mathrm{P}(2 \le F < 7)$ $=\mathrm{P}(2 \le F \le 6)$ $=\mathrm{P}(F \le 6)-\mathrm{P}(F \le 1)$ $=0.11911$ $=0.119$
(iii)	Let G be the random variable denoting the number of faulty items assembled out of 100 items. $G \sim \mathrm{B}(100, 0.04)$

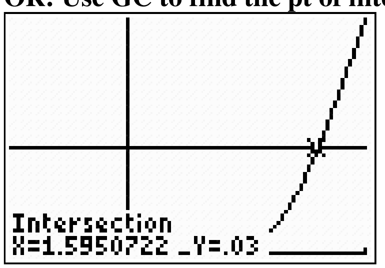
	n is large, $np = 4 < 5$, $G \sim \text{Po}(4)$ approximately $\text{P}(G' > 90) = \text{P}(G \le 9)$ $= 0.99187 = 0.992$

6(i)	
	A linear model isn't appropriate as based on the scatter diagram, as t increases, θ decreases at a decreasing rate.
(ii)	Model B is more appropriate because the temperature of the metal ball should cool down until it reaches the room temperature. It is <u>not expected to decrease indefinitely</u> in the long term.
(iii)	$y = 23.0 + \dfrac{89.1}{x}$

	(ans must be in 3 s.f. $y = 23 + \frac{89.1}{x}$ is not acceptable) r = 0.917
(iv)	At $t = 15$, $\theta = 22.9512154 + \frac{89.09716481}{t}$ $\theta = 28.9\ ^{\circ}\text{C}$ The prediction is not reliable as $t = 15$ lies outside the data range of $1 \le t \le 12$.

7(i)	P[faulty] = 0.55(0.07) + 0.35(0.04) + (1-0.55-0.35)(0.05) = 0.0575
(ii)	$P[A \mid Faulty] = P[A \cap Faulty] / P[Faulty]$ $= \frac{0.07p/100}{0.07p/100 + 0.04q/100 + 0.05\text{x}[100 - p - q]/100} = \frac{7p}{2p - q + 500}$ (shown)
(iii)	$P[\text{A} \mid Faulty] = \frac{7k}{7k + 4(100-k)}$ $P[B \mid Faulty] = \frac{4(100-k)}{7k + 4(100-k)}$ $\frac{7k}{7k+4(100-k)} > \frac{4(100-k)}{7k+4(100-k)}$ $7k > 4(100-k)$ $k > \frac{400}{11}$

	Since there are two suppliers for the component, $k<100$ Hence $100>k>\dfrac{400}{11}$

8(i)	Let X be the random variable denoting the mass of a randomly selected bar of chocolate in grams. $X\sim \mathrm{N}\left(\mu,\ \sigma^2\right)$ $\mu=\dfrac{42+48}{2}=45$ $\mathrm{P}\left(X<42\right)=0.03$ $\mathrm{P}\left(Z<\dfrac{42-45}{\sigma}\right)=0.03$ $\dfrac{42-45}{\sigma}=-1.88079361$ $\sigma=1.5951$ (shown) **OR: Use GC to find the pt of intersection**  And so $x=\sigma=1.5951$ (shown)
(ii)	$\overline{X}\sim N\left(45,\dfrac{1.5951^2}{70}\right)$

	$\mathrm{P}\left(43<\overline{X}<45\right)=0.500$
(iii)	Let Y be the random variable denoting the mass of a randomly selected bar of candy in grams. $Y \sim \mathrm{N}\left(90,\ 10^2\right)$ $X_1+X_2+X_3+X_4-2Y$ $\sim N\left(\left(45\times4\right)-\left(2\times90\right),\left(4\times1.5951^2\right)+\left(4\times100\right)\right)$ $X_1+X_2+X_3+X_4-2Y$ $\sim N\left(0,410.177376\right)$ $\mathrm{P}\left(\left\|X_1+X_2+X_3+X_4-2Y\right\|<m\right)>0.13$ $\mathrm{P}\left(-m<X_1+X_2+X_3+X_4-2Y<m\right)>0.13$ $\mathrm{P}\left(X_1+X_2+X_3+X_4-2Y<-m\right)<\dfrac{1-0.13}{2}$ (by symmetry about zero) Solving, $-m<3.314548$ $m>3.314548 \quad \Rightarrow m>3.32$

9(i)	No of ways beginning and ending with A = $=\dfrac{1.9!.1}{2!.2!}=90720$
(ii)	Group the 2 Ms and 2 As together and arrange with the other 5 single letters and then insert the 2 Ts

	No of ways = $=7!\text{x}\,{}^{8}C_{2}=141120$
(iii)	Case 1: 2 pairs of identical letters, XXYY No of ways $=\dfrac{{}^{3}C_{2}.4!}{2!.2!}=18$ Case 2: 1 pair of identical letters and 2 other different letters. No of ways = ${}^{3}C_{1}({}^{7}C_{2})(4!/2!)=756$ Case 3: 4 different letters No of ways = ${}^{8}C_{4}4!=1680$ Total no of ways = 18 +756 + 1680 = 2454

10(i)	Let D and U be the random variable denoting the number customers at the DBS's ATM and UOB's ATM respectively in 5minutes. D ~ Po(3) and D ~ Po(2.5) D + U ~ Po(5.5) Using GC,

r	$\mathrm{P}(D+U=r)$
4	0.15582

	<table><tr><td>5</td><td>0.1714</td></tr><tr><td>6</td><td>0.15712</td></tr></table> $\therefore$ The most likely total number of customers is 5.
(ii)	$\mathrm{P}(D=5 \mid D+U=5)$ $=\dfrac{\mathrm{P}(D=5 \cap D+U=5)}{\mathrm{P}(D+U=5)}$ $=\dfrac{\mathrm{P}(D=5)\cdot P(U=0)}{\mathrm{P}(D+U=5)}$ $= 0.0482828421 = 0.0483$
(iii)	Let K and L be the random variable denoting the number customers at the DBS's ATM and UOB's ATM respectively in 30 minutes. $K \sim \mathrm{Po}(18)$ and $L \sim \mathrm{Po}(15)$ $\lambda_K = 18 > 10$ and $\lambda_L = 15 > 10$ $\therefore K \sim \mathrm{N}(18, 18)$ approximately and $L \sim \mathrm{N}(15, 15)$ approximately $K - L \sim \mathrm{N}(3, 33)$approx $\mathrm{P}(K > L) = \mathrm{P}(K - L > 0)$ $\xrightarrow{cc} \mathrm{P}(K - L > 0.5)$ $= 0.668289214 = 0.668$

11(i)	Unbiased est. of population mean, $\hat{\mu} = \bar{x} = \dfrac{\sum x}{8} = \dfrac{8293}{8} = 1036.625 \approx 1040$ Unbiased est. of population variance,

	$s^2 = \frac{1}{8-1}\left[8637150 - \frac{(8293)^2}{8}\right] = 5774.125 \approx 5770$
(ii)	Let X be the random variable denoting the mass of a randomly chosen brick in grams. Assume X follows a normal distribution. $H_0 : \mu = 1000$ $H_1 : \mu > 1000$ Assuming H_0 is true, $T \sim \mathrm{t}(7)$ Value of test statistic, $t = \frac{1036.625 - 1000}{\sqrt{\frac{5774.125}{8}}} = 1.36326265$ Use GC, p-value = 0.1075116339 Since p-value = 0.108 > 0.1, we do not reject H_0. Hence, there is insufficient evidence at 5% significance level to indicate the company's claim is not valid.
(iii)	A z-test will be used instead.
	Let Y be the random variable denoting the mass of a randomly chosen brick in grams. Under H_0, $H_0 : \mu = \mu_0$ $H_1 : \mu < \mu_0$ $\overline{Y} \sim \mathrm{N}\left(\mu_0, \frac{75^2}{80}\right)$ Value of test statistic, $Z = \frac{1026 - \mu_0}{\sqrt{\frac{75^2}{80}}}$

	For H_0 to be rejected, p-value < 0.05 $\mathrm{P}\left(Z < \dfrac{1026-\mu_0}{\sqrt{\dfrac{75^2}{80}}}\right) < 0.05$ $\dfrac{1026-\mu_0}{\sqrt{\dfrac{75^2}{80}}} < -1.644853626$ $\mu_0 > 1039.792517$ Least value of $\mu_0 = 1040$ (3 s.f.)