H

CATHOLIC JUNIOR COLLEGE
General Certificate of Education Advanced Level
Higher 2
JC 2 Preliminary Examination

MATHEMATICS 9740/02

Paper 2

2 September 2014

3 hours

Additional Materials:
List of Formulae (MF15)
Graph Paper

READ THESE INSTRUCTIONS FIRST

Write your name and class on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, arrange your answers in NUMERICAL ORDER.
Place this cover sheet in front and fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages, including the cover page.

Catholic Junior College

Section A: Pure Mathematics [40 marks]

1 Sketch, on an Argand diagram, the locus of z which satisfies

$$|z-3-4\mathrm{i}|=1 \quad \text{and} \quad |z-6\mathrm{i}| \le |z-2\mathrm{i}|.$$ **[3]**

Hence, find

(i) the exact least value of $|z|$, **[1]**

(ii) the exact greatest value of arg($z-2\mathrm{i}$). **[3]**

2 The number of tiger prawns x (in thousands) in Long Xia farm at time t (in weeks) is modelled by the differential equation

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 2x - \frac{x^2}{100}.$$

(i) Show that $x = \dfrac{200}{Ae^{-2t}+1}$, where A is an arbitrary constant. **[5]**

(ii) Given that there were 120 thousands tiger prawns initially, find correct to the nearest hundred, the number of tiger prawns after 2 weeks. **[2]**

(iii) Determine the long-term implication on the number of tiger prawns in Long Xia farm. **[1]**

(iv) Sketch the graph of the solution found in part **(ii)**.
Comment on the number of tiger prawns if there were 300 thousands tiger prawns initially. **[2]**

3 Referred to the origin O, **a** and **b** are two non-zero and non-parallel vectors denoting the position vectors of the points A and B respectively.

If a point C has a position vector **c** such that **c = (a.a)a + (a.b)b**,

(i) what can you say about the points O, A, B and C ? **[1]**

(ii) Given that $OBCA$ is a parallelogram, find $|\mathbf{a}|$. **[2]**

The point U is the mid-point of OA and the point V on OB is such that OV: VB = 3: 2. The point N lies on UV such that UN: UV = 2: 3.

(iii) Find $\overrightarrow{ON}$ in terms of **a** and **b**. **[3]**

(iv) If the angle AOB is $\dfrac{\pi}{3}$, find the area of triangle OUN, giving your answer in terms of $|\mathbf{b}|$. **[4]**

4 **(a)** Find the expansion of $(4+8x)^{\frac{1}{2}}$ in ascending powers of x, up to and including the term in x^3. **[3]**

(b) Given that $y=\ln(\cos x)$, $-1<x<1$.

(i) Show that $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2+1=0$. **[3]**

(ii) By further differentiation of the result in **(i)**, find the Maclaurin's expansion for y up to and including the term in x^4. **[4]**

By using your answers in **(a)** and **(b)**, find the series expansion of $\sqrt{4+16x}-\tan x$, up to and including the term in x^3. **[3]**

Section B: Statistics [60 marks]

5 In an inspection of the items assembled on a production line, a sample of 15 items is taken and examined for faulty items.

(i) State, in context, two assumptions for the number of faulty items in the sample to be modelled by a binomial distribution. **[2]**

Assume now that the above assumptions hold and 4% of the items assembled are faulty.

(ii) Find the probability that at least 2 but fewer than 7 items are faulty. **[2]**

(iii) Another random sample of 100 items is taken. Use a suitable approximation to find the probability that more than 90 items are not faulty. **[3]**

6 A hot metal ball is left to cool down to room temperature. An experiment is conducted to observe the temperature of the metal ball as time varies. The change in successive times is shown in the following table.

Time (t minutes)	1	2	3	4	5	6	7	8	10	12
Temperature (θ °C)	99	86	64	58	43	37	29	25	24	23

(i) Draw a scatter diagram to illustrate the data, labelling the axes clearly. Comment on whether a linear model would be appropriate. **[3]**

(ii) Referring to the context of the question, state, with a reason, which of the following models is appropriate.

(A): $\theta=a+b\ln t$, where a and b are constants,

(B): $\theta=c+\frac{d}{t}$, where c and d are constants. **[1]**

(iii) For the appropriate model, calculate the equation of the regression line and the product moment correlation coefficient. **[2]**

(iv) Use your equation in part **(iii)** to predict the temperature at $t=15$. Comment on the reliability of your prediction. **[2]**

7 A manufacturing company has only 3 suppliers A, B, and C. The company buys $p\%$ and $q\%$ of components from suppliers A and B respectively and the rest from supplier C. The probability that a randomly chosen component supplied by A is faulty is 0.07. The corresponding probabilities for supplier B and C are 0.04 and 0.05 respectively.

(i) If $p = 55$ and $q = 35$, find the probability that a randomly chosen component is faulty. **[2]**

(ii) For general values of p and q, show that the probability that a randomly chosen component that is faulty was supplied by A is $\dfrac{7p}{2p-q+500}$. **[2]**

(iii) A second manufacturing company buys its components from suppliers A and B only, of which $k\%$ of them are supplied by A. Given that it is more likely that a randomly chosen component that is faulty was supplied by A, find the range of values of k. **[4]**

8 The mass of a bar of chocolate is normally distributed with mean μ grams and standard deviation σ grams. Given that 3% of the chocolates have a mass less than 42 grams and 3% have a mass more than 48 grams,

(i) state the value of μ and show that $\sigma = 1.5951$, correct to 5 significant figures. **[2]**

(ii) A random sample of 70 bars of chocolate is taken. What is the probability that the sample mean lies between 43 grams and 45 grams? **[2]**

The mass of a bar of candy is normally distributed with mean 90 grams and standard deviation 10 grams.

(iii) Given that the probability that the total mass of 4 randomly chosen bars of chocolate is within m grams of twice the mass a randomly chosen bar of candy is more than 0.13, find the range of values of m. **[4]**

9 Find the number of ways of arranging the letters of the word MATHEMATICS in which

(i) the first and the last letter are both A, **[2]**

(ii) the 2As are together and the 2Ms are together but the 2Ts must be separated, **[2]**

(iii) only 4 letters are used. **[4]**

10 In a shopping mall, there is a DBX Automated Teller Machine (ATM). The number of customers at this DBX ATM in a 5-minute interval follows a Poisson distribution with mean 3. There is another UOP ATM and the number of customers in a 5-minute interval follows an independent Poisson distribution with mean 2.5.

(i) Find the most likely total number of customers at the two ATMs in a given 5-minute interval. **[2]**

(ii) In a given 5-minute interval, there are 5 customers at the two ATMs.
Find the probability that these customers are at the DBX ATM. **[3]**

(iii) Using a suitable approximation, find the probability that, in a 30-minute interval, the number of customers at the DBX ATM is more than the number of customers at the UOP ATM. **[4]**

11 A company manufactures bricks. The mass of a brick is denoted by X grams. The masses of a random sample of 8 bricks are summarised by

$$\sum x = 8293 \text{ and } \sum x^2 = 8637150 .$$

(i) Find unbiased estimates of the population mean and variance. **[2]**

The mean mass of a brick is claimed to be 1000 grams.

(ii) Stating a necessary assumption, carry out a test at the 5% significance level on whether the mean mass of the bricks has increased. **[5]**

(iii) Suppose now that the mass of bricks follows a normal distribution with a known population variance.
What change would there be in carrying out the test using the same sample? **[1]**

Another company producing bricks claims that the mass of a brick has a mean μ_0 grams. The mass of bricks is known to have a normal distribution with standard deviation 75 grams. A random sample of 80 bricks is selected and the mean is 1026 grams.
Find, at the 5% level of significance, the least value of μ_0, correct to the nearest gram, for which there is sufficient evidence for the company to have overstated the mean mass. **[4]**

END OF PAPER