2014 DHS Year 6 Prelim Examination Paper 2

Suggested Solution

Qn	Suggested Solution
1(i)	Let P(n) be the proposition: $\displaystyle\sum_{r=1}^{n}\frac{r^2(r+1)+1}{r(r+1)}=\frac{n(n+1)^2-2}{2(n+1)}+1,\ n\in\mathbb{Z}^+.$ LHS of P(1) = $\dfrac{3}{(1)(2)}=\dfrac{3}{2}$ RHS of P(1) = $\dfrac{1(1+1)^2-2}{2(1+1)}+1=\dfrac{3}{2}=$ LHS of P(1) Hence P(1) is true. Assume P(k) is true for some $k\in\mathbb{Z}^+$, i.e. $\displaystyle\sum_{r=1}^{k}\frac{r^2(r+1)+1}{r(r+1)}=\frac{k(k+1)^2-2}{2(k+1)}+1$ To show P(k + 1) is true, i.e. $\displaystyle\sum_{r=1}^{k+1}\frac{r^2(r+1)+1}{r(r+1)}=\frac{(k+1)(k+2)^2-2}{2(k+2)}+1$ LHS of P(k + 1) $=\dfrac{k(k+1)^2-2}{2(k+1)}+1+\dfrac{(k+1)^2(k+2)+1}{(k+1)(k+2)}$ $=\dfrac{k(k+1)^2(k+2)-2(k+2)+2(k+1)^2(k+2)+2}{2(k+1)(k+2)}+1$ $=\dfrac{(k+1)^2(k+2)^2-2k-2}{2(k+1)(k+2)}+1$ $=\dfrac{(k+1)(k+2)^2-2}{2(k+2)}+1=$ RHS of P(k + 1) Hence P(k) is true $\Rightarrow$P(k+1) is true. Since P(1) is true and P(k) is true $\Rightarrow$P(k+1) is true, by mathematical induction, $\displaystyle\sum_{r=1}^{n}\frac{r^2(r+1)+1}{r(r+1)}=\frac{n(n+1)^2-2}{2(n+1)}+1$ for $n\in\mathbb{Z}^+$.
1(ii)	$\dfrac{r^2(r+1)+1}{r(r+1)}=r+\dfrac{1}{r(r+1)}=r+\dfrac{1}{r}-\dfrac{1}{r+1}$ By cover-up rule, $A=1$, $B=-1$ i.e. $\displaystyle\sum_{r=1}^{n}\frac{r^2(r+1)+1}{r(r+1)}=\sum_{r=1}^{n}\left(r+\frac{1}{r}-\frac{1}{r+1}\right)$

	$\sum_{r=1}^{n}\frac{r^2(r+1)+1}{r(r+1)}=\sum_{r=1}^{n}r+\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$ $=\sum_{r=1}^{n}r+\begin{bmatrix}1-\cancel{\frac{1}{2}}\\+\cancel{\frac{1}{2}}-\cancel{\frac{1}{3}}\\\vdots\\+\cancel{\frac{1}{n}}-\frac{1}{n+1}\end{bmatrix}$ $=\sum_{r=1}^{n}r+\left(1-\frac{1}{n+1}\right)$ $\sum_{r=1}^{n}r=\sum_{r=1}^{n}\frac{r^2(r+1)+1}{r(r+1)}-\sum_{r=1}^{n}\left(\frac{1}{r}-\frac{1}{r+1}\right)$ $=\frac{n(n+1)^2-2}{2(n+1)}+1-\left(1-\frac{1}{n+1}\right)$ $=\frac{n(n+1)}{2}-\frac{1}{n+1}+1-1+\frac{1}{n+1}$ $=\frac{n(n+1)}{2}$

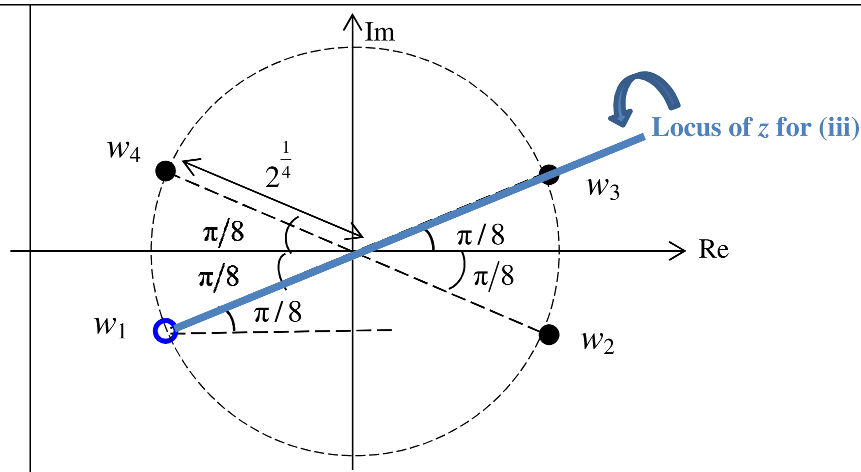
Qn	Suggested Solution
2(i)	$k=\left(\frac{1}{2}x^2\sin\frac{\pi}{3}+hx\right)(3x)$ $=\frac{3\sqrt{3}}{4}x^3+3hx^2$ $\therefore h=\frac{1}{3x^2}\left(k-\frac{3\sqrt{3}}{4}x^3\right)=\frac{k}{3x^2}-\frac{\sqrt{3}}{4}x$ $A=2\left(\frac{1}{2}x^2\sin\frac{\pi}{3}\right)+2(3x^2)+2(hx)+2(3hx)$ $=\frac{\sqrt{3}}{2}x^2+6x^2+8hx$ $=\frac{\sqrt{3}}{2}x^2+6x^2+8x\left(\frac{k}{3x^2}-\frac{\sqrt{3}}{4}x\right)$ $=\frac{\sqrt{3}}{2}x^2+6x^2+\frac{8k}{3x}-2\sqrt{3}x^2$ $=6x^2-\frac{3\sqrt{3}}{2}x^2+\frac{8k}{3x}$ (shown)

	$\therefore \frac{dA}{dx} = 12x - 3x\sqrt{3} - \frac{8k}{3x^2}$ For stationary values, $\frac{dA}{dx} = 12x - 3x\sqrt{3} - \frac{8k}{3x^2} = 0$ $12x - 3x\sqrt{3} - \frac{8k}{3x^2} = 0$ $9x^3\left(4-\sqrt{3}\right) = 8k$ $x^3 = \frac{8k}{9\left(4-\sqrt{3}\right)}$ $\therefore x = \left(\frac{8k}{9\left(4-\sqrt{3}\right)}\right)^{\frac{1}{3}}$ $\frac{d^2A}{dx^2} = 12 - 3\sqrt{3} + \frac{16k}{3x^3}$ Since $x^3 > 0$, $k > 0$, $12 - 3\sqrt{3} > 0$ <u>Alternative</u> $\frac{d^2A}{dx^2} = 12 - 3\sqrt{3} + \frac{16k}{3x^3}$ $= 12 - 3\sqrt{3} + \frac{16k}{3}\left(\frac{9\left(4-\sqrt{3}\right)}{8k}\right)$ $= 12 - 3\sqrt{3} + 6\left(4-\sqrt{3}\right)$ $= 36 - 9\sqrt{3} > 0$ $\therefore$ area A is a minimum.
2(ii)	Using $k = 360$ and $A = 300$, $300 = 6x^2 - \frac{3\sqrt{3}}{2}x^2 + \frac{8(360)}{3x}$ $1800x = 36x^3 - 9\sqrt{3}x^3 + 5760$ $x^3 - 88.185x + 282.19 = 0$ From GC, since $x > 0$, $x = 3.8442$ or $x = 6.8587$ When $x = 3.8442$, $h = \frac{360}{3(3.8442)^2} - \frac{\sqrt{3}}{4}(3.8442) = 6.46.$

	When $x = 6.8587$, $h = \dfrac{360}{3(6.8587)^2} - \dfrac{\sqrt{3}}{4}(6.8587) = -0.419$ (rej. $\because\ h > 0$) $\therefore\ x = 3.84,\ h = 6.46.$

Qn	Suggested Solution
3(i)	For $w^4 - 2w^2 + 2 = 0$, $w^2 = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2} = 1 \pm \mathrm{i}$ $w^2 = 1 + \mathrm{i} = \sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ $= \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4} + 2k\pi\right)}$ $\therefore w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8} + k\pi\right)}$, $k = 0, -1$ $\therefore w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{7\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}$ Since the coefficients of the equation are real, the conjugates of the above roots of w are also roots for the equation. $\therefore w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{7\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{7\pi}{8}\right)}$
	<u>Alternatively</u> For $w^4 - 2w^2 + 2 = 0$, $w^2 = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2} = 1 \pm \mathrm{i}$ $w^2 = 1 + \mathrm{i}$ or $w^2 = 1 - \mathrm{i}$ $= \sqrt{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$ $\quad$ $= \sqrt{2}\mathrm{e}^{-\mathrm{i}\frac{\pi}{4}}$ $= \sqrt{2}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{4} + 2k\pi\right)}$ $\quad$ $= \sqrt{2}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{4} + 2k\pi\right)}$ $\therefore w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8} + k\pi\right)}$, $k = 0, -1$ or $w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{8} + k\pi\right)}$, $k = 0, 1$ $\therefore w = 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{7\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(-\frac{\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{\pi}{8}\right)}, 2^{\frac{1}{4}}\mathrm{e}^{\mathrm{i}\left(\frac{7\pi}{8}\right)}$ **[Note: Students ought to recognize that roots of *w* must be in conjugate pairs due to real coefficients in the equation.]**

3(ii)



[Note: Students ought to recognize that since the roots of *w* are in conjugate pairs, they will be reflections of one another in the *x*-axis.]

3(iii)

Since the locus is a half-line from the point representing w_1 and angled at $\pi/8$ from the positive Re-axis direction, it will pass through the origin and eventually the point representing w_3 since $\arg(w_3) = \pi/8$

Alternative Method

$$\arg(2^{1/4}e^{i(\pi/8)} - 2^{1/4}\left[\cos(-7\pi/8) + i\sin(-7\pi/8)\right])$$

$$=\arg\left(2^{1/4}\left[e^{i(\pi/8)} - e^{i(-7\pi/8)}\right]\right)$$

$$=\arg\left(2^{1/4}e^{i(\pi/8)}\left[1 - e^{i(-\pi)}\right]\right)$$

$$=\arg\left(2^{1/4}e^{i(\pi/8)}\left[1-(-1)\right]\right)$$

$$=\arg\left(2^{5/4}e^{i(\pi/8)}\right)$$

$$=\frac{\pi}{8}$$

Alternatively,

substitute $z = w_3$ into LHS of $\arg(z - w_1) = \dfrac{\pi}{8}$

$$\arg(w_3 - w_1)$$

$$= \arg(-w_1 - w_1) \quad \text{(Since } w_3 = -w_1\text{, rotation of } \pi \text{ about origin)}$$

$$= \arg(-2w_1)$$

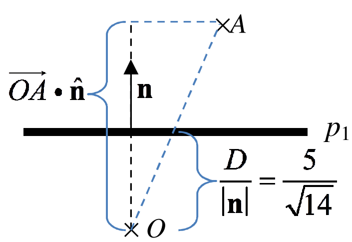
$$= \arg(-2) + \arg(w_1)$$

$$= \pi - \frac{7\pi}{8}$$

$$= \frac{\pi}{8}$$

***Locus drawn in (ii)**

	Cartesian Equation of locus: $y = x\tan\left(\frac{\pi}{8}\right), \quad x > 2^{1/4}\cos\left(-\frac{7\pi}{8}\right)$

Qn	Suggested Solution
4(i)	$l : \underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ Let the acute angle between l and p_1 be θ. $\cos(90° - \theta) = \sin\theta = \left\| \frac{\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}}{\sqrt{2}\sqrt{14}} \right\| = \frac{1}{\sqrt{28}}$ $\theta = 10.9°$
4(ii)	<u>Method 1</u> $(0,5,0)$ is a point on p_1. Perpendicular distance from the point A to p_1 $= \frac{\left\| \left[\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 5 \\ 0 \end{pmatrix} \right] \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\|}{\sqrt{14}}$ $= \frac{\left\| \begin{pmatrix} 2 \\ -3 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\|}{\sqrt{14}} = \frac{3}{\sqrt{14}}$ <u>Method 2</u> Perpendicular distance from the point A to p_1 $= \frac{\left\| 5 - \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\|}{\sqrt{14}}$ $= \frac{\|5-8\|}{\sqrt{14}} = \frac{3}{\sqrt{14}}$ 

	Method 3 Let F be the foot of perpendicular and it lies on both l_{AF} and p_1. $l_{AF}: \underset{\sim}{r} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}, \ \mu \in \mathbb{R}$ $\left[\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right] \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 5$ $\Rightarrow \mu = -\frac{3}{14}$ $\overrightarrow{OF} = \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \left[-\frac{3}{14} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right]$ $\overrightarrow{AF} = \overrightarrow{OF} - \overrightarrow{OA} = -\frac{3}{14} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$ $\left\| \overrightarrow{AF} \right\| = \left\| -\frac{3}{14} \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \right\| = \frac{3\sqrt{14}}{14}$
4(iii)	$l: \underset{\sim}{r} = \begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ **To find b:** Method 1 Direction vector of l is **perpendicular** to normal vector of p_3, $\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} = 0 \Rightarrow -1 + b = 0 \Rightarrow b = 1$ Method 2 $\begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} = k \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\begin{pmatrix} 1-4b \\ 1-b \\ 3 \end{pmatrix} = k \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ $\Rightarrow k = 3, \quad b = 1$ **To find c:** Method 1 Since l lies on p_2,

$$\begin{pmatrix} 2 \\ -1 \\ a \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} = 6$$

$$2 - 4(-1) + a = 6 \Rightarrow a = 0$$

Since l lies on p_3,

$$\begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = c$$

$$2 - (-1) + b(0) = c \Rightarrow c = 3$$

<u>Method 2</u>

Since l lies on p_2,

$$\begin{pmatrix} 2-\lambda \\ -1 \\ a+\lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -4 \\ 1 \end{pmatrix} = 6$$

$$\Rightarrow 6 + a = 6 \Rightarrow a = 0$$

Since l lies on p_3,

$$\begin{pmatrix} 2-\lambda \\ -1 \\ \lambda \end{pmatrix} \bullet \begin{pmatrix} 1 \\ -1 \\ b \end{pmatrix} = c$$

$$(3-c) + (b-1)\lambda = 0$$

Since the equation is **<u>always true regardless of λ,</u>**

$3 - c = 0 \Rightarrow c = 3$ &

$b - 1 = 0 \Rightarrow b = 1$

4(iv)

Let M be the midpoint of A and B.

$$\overrightarrow{OM} = \frac{\overrightarrow{OA} + \overrightarrow{OB}}{2} = \frac{1}{2}\left[\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \overrightarrow{OB}\right]$$

$$\frac{1}{2}\left[\begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} + \overrightarrow{OB}\right] \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 5$$

$$\Rightarrow \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} + \overrightarrow{OB} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 10$$

$$\Rightarrow \overrightarrow{OB} \bullet \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} = 10 - 8 = 2$$

$\therefore$ A cartesian equation for the locus of B is

$3x + y + 2z = 2.$

Note: Locus is a plane parallel to p_1

Qn	Suggested Solution
5(a)	Advantages of stratified sampling over quota sampling - More likely to give a representative sample of the people attending the event as this method ensures an adequate sample size for each of the two sub-groups - The stratified sample is a random sample. It is unlike quota sampling which produces a biased sample by using any method of convenience to select the people for each sub-group. A stratified sample is difficult to carry out as it would be difficult to obtain the sampling frame prior to the event.
(b)	**Remark to students:** Please note that even though sampling frame was not readily available before the event, by standing at a common place that all people will pass through (eg: exit of the venue), the surveyor would still be able to have access to the entire sampling frame for systematic sampling. To obtain a systematic sample of 1% of the population of people attending the event: • Select a random number between 1 to 100 to get a random starting point e.g. 10 • From the start to the end of the one-day event, sample every 100^{th} person at the exit of the venue, starting from the 10^{th} person, i.e. 10, 110, 210, 310,…

Qn	Suggested Solution
6(i)	Probability = P(drawing from box A,B,A,B,A,B) =P(drawing R,W,R,W,R) $=\left(\frac{5}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{9}\right)\left(\frac{5}{9}\right)\left(\frac{3}{8}\right)$ $=\frac{1}{36}$
6(ii)	Let event E be the event that he drew from box B on the fourth draw. Let event F be the event that he has not drawn from box C from the first to his sixth draw, including the sixth draw. $\mathrm{P}(\mathrm{F}\cap\mathrm{E})$ $=\frac{1}{36}$ $\mathrm{P}(\mathrm{E})$ = P(drawing from A,B,C,B or A,B,A,B or A,C,A,B) = P(drawing R,R,W or R,W,R or W,R,R) $=\left(\frac{5}{10}\right)\left(\frac{4}{10}\right)\left(\frac{7}{10}\right)+\left(\frac{5}{10}\right)\left(\frac{6}{10}\right)\left(\frac{4}{9}\right)+\left(\frac{5}{10}\right)\left(\frac{3}{10}\right)\left(\frac{5}{9}\right)$ $=\frac{107}{300}$

$$P(F|E)$$
$$=\frac{P(F\cap E)}{P(E)}$$
$$=\frac{P(\text{drawing from box A,B,A,B,A,B})}{P(\text{drawing from box B on fourth draw})}$$
$$=\frac{\frac{1}{36}}{\frac{107}{300}}$$
$$=\frac{25}{321}$$

Qn	Suggested Solution

7(i)

There are 8! ways to arrange the 8 girls. There are 9 possible spaces where boys can be slotted in. Choose 4 spaces out of 9. There are 4! ways to arrange the boys.

↓ G_1 ↓ G_2 ↓ G_3 ↓ G_4 ↓ G_5 ↓ G_6 ↓ G_7 ↓ G_8 ↓

P(all boys are separated) = $\frac{8!\,{}^9C_4\,4!}{12!}=\frac{14}{55}$ or 0.255

7(ii)

Choose 2 out of 8 girls to be on both sides of the particular boy. There are 2! ways to arrange the 2 girls. Taking GBG as one group, there are 10! ways to arrange the group and 9 other students.

GBG G G G G G G B B B

P(a particular boy is between 2 girls)

$= \frac{{}^8C_2\,2!\,10!}{12!}=\frac{14}{33}$ or 0.424

7(iii)

Taking the boys as one group, there are 4! ways to arrange the boys. Taking the boys group as reference point, there are 8! ways to arrange the rest of the 8 girls.

P(all boys next to one another in a circle)

$= \frac{4!\,8!}{(12-1)!}=\frac{4}{165}$ or 0.0242

7(iv)

<u>Method 1</u>

P(at least 1 student from each race)

$= 1-P(\text{Chinese and Malay only or Chinese and Indian only})$

$$= 1 - \frac{{}^{10}C_7 + {}^{8}C_7}{{}^{12}C_7}$$

$$= 1 - \frac{16}{99}$$

$$= \frac{83}{99} \text{ or } 0.838$$

Method 2

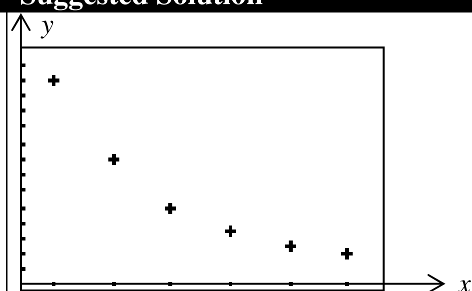
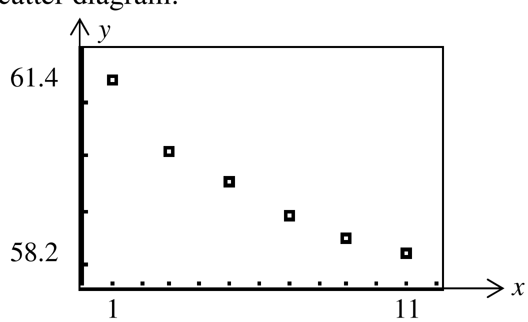
Let C, I, M to denote a Chinese, Indian, and Malay student respectively.

P(at least 1 student from each race)

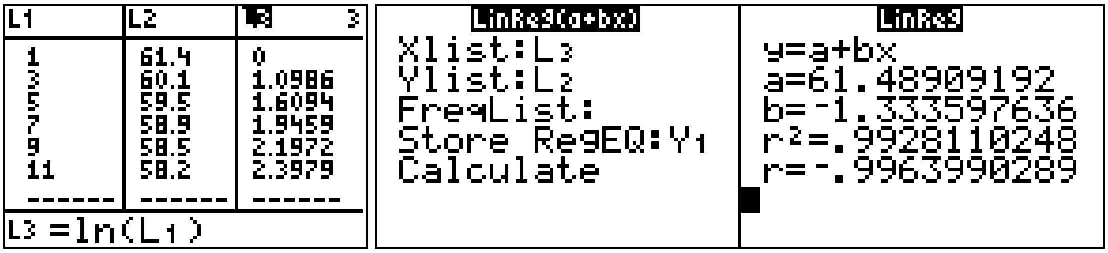
= 1 – P(6C 1M, 6C 1I, 5C 2I, 5C 2I, 4C 3M, 3C 4M)

$$= 1 - \frac{\binom{6}{6}\binom{4}{1} + \binom{6}{6}\binom{2}{1} + \binom{6}{5}\binom{2}{2} + \binom{6}{5}\binom{4}{2} + \binom{6}{4}\binom{4}{3} + \binom{6}{3}\binom{4}{4}}{\binom{12}{7}}$$

$$= \frac{83}{99}$$

Qn	Suggested Solution
8(i)	
8(ii)	Scatter diagram: 
(iii)	$y = a + b\ln x$ is the better model. **Reason:** y is decreasing at a **decreasing rate** as x increases.
(iv)	**Interpretation:** Value of a represents Amy's predicted weight one month after she began her healthy eating lifestyle and exercise regime. r = – 0.996 (3sf)

Useful Screenshots for reference:



[**Remark:** Those who indicated that the linear model is the better model in (iii) will be given marks accordingly.]

<u>For those who chose $y = c + dx$ in (iii):</u>

Value of c represents <u>Amy's predicted initial weight</u> before she began her healthy eating lifestyle and exercise regime

$r = -0.967$

(v) Equation of regression line:

$y = 61.489 - 1.3336\ln x$

When $y = 55$:

$55 = 61.489 - 1.3336\ln(x)$

$x = 129.77$ (5sf)

She will reach a weight of 55kg in the <u>130th month</u> after she started.

<u>For those who chose $y = c + dx$ in (iii):</u>

Equation of line as $y = 61.268 - 0.30571x$

Substituting $y = 55$ into equation to obtain $x = 20.503$

She will reach a weight of 55kg in the <u>21st month</u> after she started.

(vi) **<u>For any model chosen in (iii):</u>**

As $x \to \infty, y \to -\infty$.

Thus, this implies that Amy's weight will decrease to a **negative value over time**, which is unrealistic.

Qn	Suggested Solution
9(i)	Let X be the sleeping hours of a randomly chosen baby who drinks BabyGrow.
	$X \sim \mathrm{N}\left(8.0,\ \sigma^2\right)$
	Let Y be the sleeping hours of a randomly chosen baby who drinks InfanGrow.
	$Y \sim \mathrm{N}\left(6.5,\ 0.795^2\right)$
	$\mathrm{P}(X < 9) = 0.85$

$$\mathrm{P}\left(Z < \frac{9-8}{\sigma}\right) = 0.85$$

$$\frac{9-8}{\sigma} = 1.0364$$

$$\sigma = 0.96488$$

$$= 0.965 \text{ (3 s.f.)}$$

9(ii)

Let W be the number of babies who slept more than 7 hours, out of 12 babies.

$\mathrm{P}(X > 7) = \mathrm{P}(X < 9) = 0.85$ (by symmetry)

$W \sim \mathrm{B}(12, P(X > 7))$

$\Rightarrow W \sim \mathrm{B}(12, 0.85)$

$\mathrm{P}(W \geq 10)$

$= 1 - \mathrm{P}(W \leq 9)$

$= 1 - 0.26418$

$= 0.73582 = 0.736$ (3 s.f.)

9(iii)

Let $T = X_1 + X_2 + X_3 \sim \mathrm{N}\left(3\times 8,\ 3\times 0.96488^2\right)$

$\Rightarrow T \sim \mathrm{N}(24,\ 2.7930)$

$4Y \sim \mathrm{N}\left(4\times 6.5,\ 4^2\times 0.795^2\right) \Rightarrow 4Y \sim \mathrm{N}(26,\ 10.112)$

$\therefore T - 4Y \sim \mathrm{N}(24-26,\ 2.7930+10.112)$

$\Rightarrow T - 4Y \sim \mathrm{N}(-2,\ 12.905)$

$\mathrm{P}(T - 4Y > 1)$

$= 0.20183 = 0.202$ (3 s.f.)

9(iv)

$\mathrm{P}(Y < 5.5) = 0.10422$

Expected number of babies on InfanGrow who will develop obesity

$= 20\% \times 300 \times \mathrm{P}(Y < 5.5)$

$= 6.25$ (3 s.f.)

Qn	**Suggested Solution**

10 (i)

Let X = number of Math appointments in 5-day period

$X \sim \mathrm{Po}(5(1.8)) \Rightarrow X \sim \mathrm{Po}(9)$

$\mathrm{P}(X \leq 6) = 0.207$ (3 sf, by GC)

Use GC Poisson pdf to generate listing:

r	$P(X = r)$
7	0.11712
8	0.13176
9	0.13176
10	0.11858

The most probable numbers = **8** and **9**.

Recall fact**:** For Poisson distribution where λ is an integer, **Mode =** $(\lambda - 1)$ and λ

10 (ii)	Let D denote the number of days out of 30, that there are exactly 3 Math appointments a day. $D \sim \mathrm{B}(30, 0.161)$ Find minimum n such that $\mathrm{P}(D \le n) \ge 0.95$ Use GC Binomcdf listing: $\mathrm{P}(D \le 7) = 0.903 \le 0.95$ $\mathrm{P}(D \le 8) = 0.958 \ge 0.95$ $\mathrm{P}(D \le 9) = 0.984$ $\therefore$ least $n = 8$
	The **mean number** of appointments **may not be constant from day to day** because **there may be more appointments near exam period and few or none during the non-exam period.**
10 (iii)	Let M and S denote the number of Math & Science appointments respectively in 30 -day period. $M \sim \mathrm{Po}(30(1.8)) \Rightarrow M \sim \mathrm{Po}(54)$ $S \sim \mathrm{Po}(30(2.2)) \Rightarrow S \sim \mathrm{Po}(66)$ Since $\lambda_M = 54 > 10$ and $\lambda_S = 66 > 10$, hence $M \sim \mathrm{N}(54, 54)$ approx and $S \sim \mathrm{N}(66, 66)$ approx Thus, $\boldsymbol{S - M \sim \mathrm{N}(12, 120)}$ approximately $\mathrm{P}(S - M > 12)$ $= \mathrm{P}(S - M > 12.5)$ with **continuity correction** $=$ 0.482 (3 sf)

Qn	Suggested Solution
11(i)	An unbiased estimate is an estimate in which the expectation of the estimator is equal to the population parameter.
11(ii)	Unbiased estimate for population mean $= \dfrac{\sum(x-18)}{50} + 18$ $= 21.306$ Unbiased estimate of population variance σ^2 $= s^2 = \dfrac{1}{49}\left[\sum(x-18)^2 - \dfrac{\left(\sum(x-18)\right)^2}{50}\right]$ $= 6.7351$ $= 6.74$ (3 s.f.)

11(iii)

Let X be the mass of each bag of beans produced in Factory A.

To test $\mathrm{H}_0 : \mu = 22$

Against $\mathrm{H}_1 : \mu \neq 22$

Conduct a Two-tailed test at 5% level of significance:

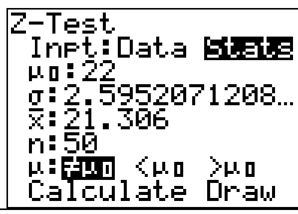
Under H_0, since $n = 50$ is large, by Central Limit Theorem,

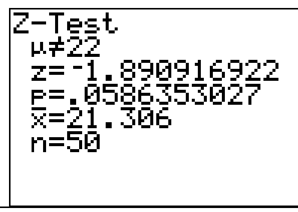
$$\overline{X} \sim \mathrm{N}\left(22, \frac{6.7351}{50}\right) \text{ approximately}$$

Using a Z-test,

p-value = 0.0586 > 0.05

Since p-value > 0.05, we do not reject H_0 and conclude that there is insufficient evidence at the 5% level of significance that the claim is not valid / that the mean mass of each bag of beans is not 22 kg.





11(iv)

p-value is the lowest level of significance for which the null hypothesis of the mean mass of the bag of beans of 22 kg will be rejected.

Let Y be the mass of each bag of beans produced in Factory A.

Objective: To test if $\mu = \mu_0$ is an overstatement.

To test $\mathrm{H}_0 : \mu = \mu_0$

Against $\mathrm{H}_1 : \mu < \mu_0$

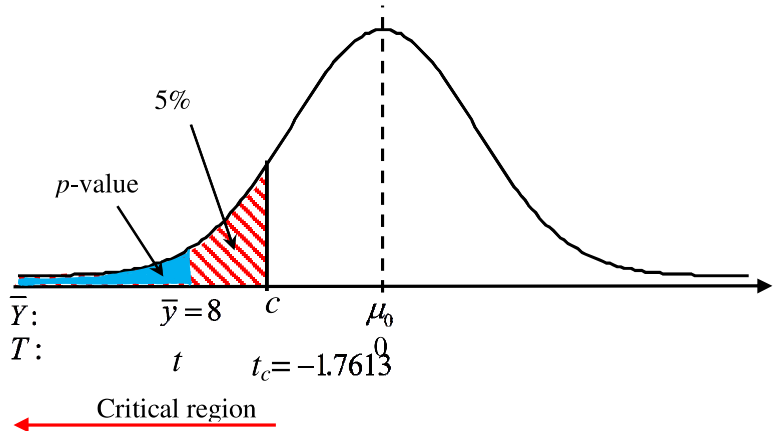
Under H_0,

$$T = \frac{\overline{Y} - \mu_0}{\sqrt{\frac{s^2}{n}}} = \frac{\overline{Y} - \mu_0}{\frac{0.2}{\sqrt{15}}} \sim t(14)$$

Using test statistic, $t = \dfrac{8 - \mu_0}{\frac{0.2}{\sqrt{15}}}$,

since there is sufficient evidence at 5% level of signficance,

$\Rightarrow$ H_0 is rejected



Reject H_0: $\bar{y} < c$, $t < t_c$

Method 1:

$\mathrm{P}(\bar{Y} < 8) < 0.05$

$$\mathrm{P}\left(T < \frac{8-\mu_0}{\frac{0.2}{\sqrt{15}}}\right) < 0.05$$

$$\frac{8-\mu_0}{\frac{0.2}{\sqrt{15}}} < -1.7613$$

$\mu_0 > 8.09$

$\therefore$ Set of values of $\mu_0 = \{\mu_0 \in \mathbb{R} : \mu_0 > 8.09\}$

Method 2:

$\Rightarrow$ critical region : $t < -1.7613$

$$\Rightarrow \quad \frac{8-\mu_0}{\frac{0.2}{\sqrt{15}}} < -1.7613$$

$\Rightarrow \quad \mu_0 > 8.09$

$\therefore$ Set of values of $\mu_0 = \{\mu_0 \in \mathbb{R} : \mu_0 > 8.09\}$