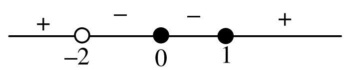
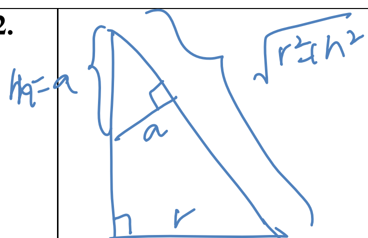
Solution		
1.	$\dfrac{x^2(x-1)}{x+2} \geq 0$ Apply **test-point** method,  $x \geq 1$ or $x < -2$ or $x = 0$	
	$\dfrac{x^2(\lvert x\rvert-1)}{\lvert x\rvert+2} \geq 0$ Replace x by $\lvert x\rvert$: $\lvert x\rvert \geq 1$ or $\lvert x\rvert < -2$ or $\lvert x\rvert = 0$ $\Rightarrow x \leq -1$ or $x \geq 1$ or $x = 0$	

2.	 $\dfrac{r}{\sqrt{r^2+h^2}} = \dfrac{a}{h-a}$	

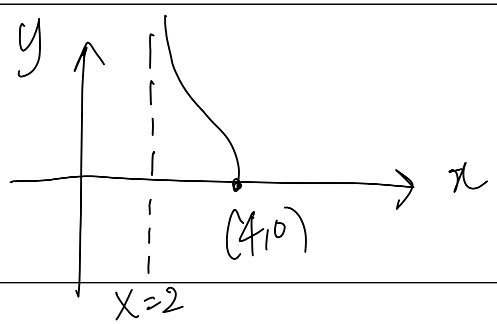
	$(h-a)(r)=a\sqrt{r^2+h^2}$ $\Rightarrow (h-a)^2(r)^2=a^2(r^2+h^2)$ $\mathrm{r}^2(h^2-2ah+a^2-a^2)=a^2h^2$ $\mathrm{r}^2=\dfrac{a^2h^2}{h^2-2ah}=\dfrac{a^2h}{h-2a}$ $V=\dfrac{1}{3}\pi r^2h=\dfrac{1}{3}\pi\left(\dfrac{a^2h}{h-2a}\right)h$ $V=\dfrac{\pi a^2h^2}{3(h-2a)}$ $\dfrac{dV}{dh}=\dfrac{\pi a^2}{3}\cdot\dfrac{(h-2a)(2h)-h^2}{(h-2a)^2}$ $=\dfrac{\pi a^2}{3}\cdot\dfrac{h^2-4ah}{(h-2a)^2}$ When V is maximum, $\dfrac{dV}{dh}=0$ $h=0\,(rej),\quad h=4a$	

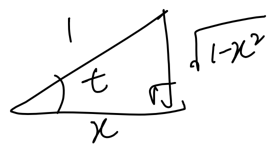
3(i)	Let d be the common difference. $2+(7-1)d=1.82$ $\qquad$ $6d=-0.18$ $d=-0.03$ Hence the common difference is negative 0.03 metres.	
(ii)	$T_n=2+(n-1)(-0.03)=1.10$ $(n-1)(-0.03)=-0.90$ $(n-1)=30$	

	$n=31$	
(iii)	$T_{31}=2r^{(31-1)}=1.10$ $r^{30}=0.55$ $r=0.980269...\approx 0.9803 \quad \text{(4 d.p.)}$	
(iv)	Original cost $=(20)\left(\frac{31}{2}\right)\left[2(2)+(31-1)(-0.03)\right]$ $=\$961.00$ Modified cost $=(20)\left(\frac{2\left(1-0.9802693482^{31}\right)}{1-0.9802693482}\right)$ $=\$934.29$ Difference $=\$961.00-\$934.29=\$26.71$	

4.	$\sum_{n=2}^{N}\frac{1}{n^2-1}=\sum_{n=2}^{N}\left[\frac{1}{2}\frac{1}{(n-1)}-\frac{1}{2}\frac{1}{(n+1)}\right]$ $=\frac{1}{2}\left[\begin{array}{l}\left(1-\frac{1}{3}\right)+\left(\frac{1}{2}-\frac{1}{4}\right)+\left(\frac{1}{3}-\frac{1}{5}\right)+....+\left(\frac{1}{N-3}-\frac{1}{N-1}\right)+\\ \left(\frac{1}{N-2}-\frac{1}{N}\right)+\left(\frac{1}{n-1}-\frac{1}{N+1}\right)\end{array}\right]$ $=\frac{1}{2}\left[1+\frac{1}{2}-\frac{1}{N}-\frac{1}{N+1}\right]$ $=\frac{1}{2}\left[\frac{3}{2}-\frac{1}{N}-\frac{1}{N+1}\right]$	

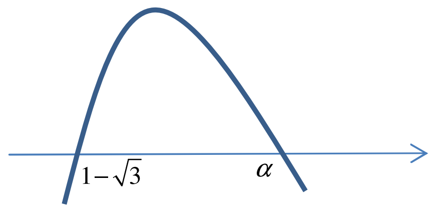
	as $N \to \infty, \sum_{n=2}^{N} \frac{1}{n^2-1} = \frac{1}{2}\left[\frac{3}{2} - \frac{1}{N} - \frac{1}{N+1}\right]$ becomes $\sum_{n=2}^{\infty} \frac{1}{n^2-1} = \frac{1}{2}\left[\frac{3}{2}\right] = \frac{3}{4}$ *Note* $\sum_{n=2}^{\infty} \frac{1}{n^2} < \sum_{n=2}^{\infty} \frac{1}{n^2-1}$ $1+\sum_{n=2}^{\infty} \frac{1}{n^2} < 1+\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ $\therefore \sum_{n=1}^{\infty} \frac{1}{n^2} < 1+\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ $\therefore \sum_{n=1}^{\infty} \frac{1}{n^2} < 1+\frac{3}{4} = \frac{7}{4}$	

5(ai)	$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{-2\sin t} = \frac{1}{-2\sin t\cos^2 t}$ Since $\frac{dy}{dx} < 0$ for all $t \in \mathbb{R}$ $\therefore$ Curve C has no stationary point.	
(aii)		

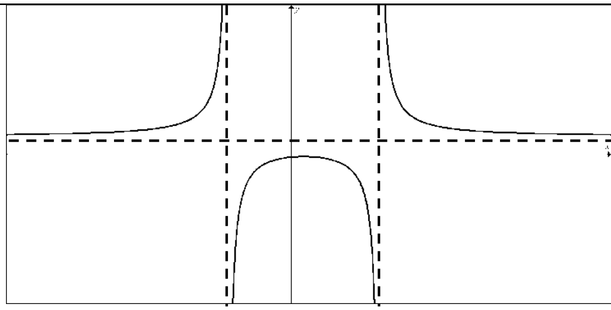
(aiii)	At $t=\dfrac{\pi}{4}$, $x=2+\sqrt{2}$, $y=1$, $\dfrac{dy}{dx}=-\sqrt{2}$ $y-1=-\sqrt{2}(x-2-\sqrt{2})$ $y=-\sqrt{2}x+2\sqrt{2}+3$	
	$M\ (\cos t, \frac{1}{2}\tan t)$ Let $x=\cos t$, $y=\frac{1}{2}\tan t$.  $y=\frac{1}{2}\left(\frac{\sqrt{1-x^2}}{x}\right)$ OR using trigo identity, $1+\tan^2 t=\sec^2 t$ $1+4y^2=\frac{1}{x^2}$ $y=\frac{1}{2}\left(\frac{\sqrt{1-x^2}}{x}\right)$, since $y>0$ for $0\le t<\frac{\pi}{2}$.	

6(a)		
	Let P_n denote the proposition $\sum_{r=1}^{n} r\ln\left(\frac{r+1}{r}\right) = \ln\frac{(n+1)^n}{n!}$ for $n \in \mathbb{Z}^+$ When $n = 1$, LHS = $\sum_{r=1}^{1} r\ln\left(\frac{r+1}{r}\right) = 1\times\ln\frac{(1+1)}{1} = \ln 2$ RHS $= \ln\frac{(1+1)^1}{1!} = \ln 2$ $\therefore$ P_1 is true Assume that P_k is true for some $k \in \mathbb{Z}^+$, i.e. $\sum_{r=1}^{k} r\ln\left(\frac{r+1}{r}\right) = \ln\frac{(k+1)^k}{\text{k}!} = k\ln(k+1) - \ln k!$ To prove P_{k+1} is also true i.e. $\sum_{r=1}^{k+1} r\ln\left(\frac{r+1}{r}\right) = \ln\frac{(k+2)^{k+1}}{(k+1)!}$ $= (k+1)\ln(k+2) - \ln(k+1)!$ LHS = $\sum_{r=1}^{k+1} r\ln\left(\frac{r+1}{r}\right)$ $= \sum_{r=1}^{k} r\ln\left(\frac{r+1}{r}\right) + (k+1)\left[\ln(k+2) - \ln(k+1)\right]$ $= k\ln(k+1) - \ln(k!) + (\text{k+1})\left[\ln(k+2) - \ln(k+1)\right]$ $= \left[k - (k+1)\right]\ln(k+1) - \ln(k!) + (k+1)\ln(k+2)$ $= -\ln(k+1) - \ln(k!) + (k+1)\ln(k+2)$ $= -\ln(k+1)! + (k+1)\ln(k+2)$ Hence P_{k+1} is true	

	Since P_1 is true and P_k is true $\Rightarrow P_{k+1}$ is true, by mathematical induction, P_n is true for all $n \in \mathbb{Z}^+$ **Alternative** To prove P_{k+1} is also true i.e. $\sum_{r=1}^{k+1} r\ln\left(\frac{r+1}{r}\right)=\ln\frac{(k+2)^{k+1}}{(k+1)!}$ LHS = $\sum_{r=1}^{k+1} r\ln\left(\frac{r+1}{r}\right)$ $=\sum_{r=1}^{k} r\ln\left(\frac{r+1}{r}\right)+(k+1)\ln\left(\frac{k+2}{k+1}\right)$ $=\ln\frac{(k+1)^k}{(k)!}+(k+1)\ln\left(\frac{k+2}{k+1}\right)$ $=\ln\left[\frac{(k+1)^k}{(k)!}\times\left(\frac{k+2}{k+1}\right)^{(k+1)}\right]$ $=\ln\left[\frac{(k+1)^k(k+2)^{k+1}}{(k!)(k+1)^{k+1}}\right]$ $=\ln\left[\frac{(k+2)^{k+1}}{(k+1)!}\right]$=RHS	
6b(i)	As $n\to\infty$, $x_n\to\alpha$, $\alpha=\frac{3\alpha+2}{\alpha+1}$ $\therefore \alpha^2+\alpha=3\alpha+2$ $\alpha^2-2\alpha-2=0$ $\Rightarrow \alpha=1+\sqrt{3}$ or $1-\sqrt{3}$ (rejected $\because x_n>0$)	

(ii) **(iii)**	Using GC , when $x_1 = 3.5$. the sequence is decreasing and converges to α $x_{n+1} - x_n = \dfrac{3x_n + 2}{x_n + 1} - x_n$ $= \dfrac{3x_n + 2 - x_n^{\ 2} - x_n}{x_n + 1}$ $= \dfrac{-\left(x_n^{\ 2} - 2x_n - 2\right)}{x_n + 1}$ Since $x_n > 0$, $x_n + 1 > 0$. Consider the graph of $y = -\left(x_n^{\ 2} - 2x_n - 2\right)$  If $x_n > \alpha \Rightarrow x_n - \alpha < 0$ $\therefore x_{n+1} - x_n < 0 \Rightarrow x_{n+1} < x_n$ Hence the sequence is decreasing and converging to α which support the answer in part (ii).	

8(i)	$\mathrm{f}\left(x\right) = \dfrac{x^2 - 2x + 5}{x^2 - 2x - 35}$ $= 1 + \dfrac{40}{x^2 - 2x - 35}$ $= 1 + \dfrac{10}{3(x-7)} - \dfrac{10}{3(x+5)}$	

	$f'(x)=\frac{10}{3(x+5)^2}-\frac{10}{3(x-7)^2}=\frac{80-80x}{(x+5)^2(x-7)^2}$ *<u>(Alternatives include direct differentiation using Quotient rule or Product rule.)</u>* At stationary point, $80-80x=0$ Solving, $x=1$, $y=-\frac{1}{9}$ $\therefore\left(1,-\frac{1}{9}\right)$	
(ii)	 Asymptotes: $y=1$, $x=-5$, $x=7$ Axial intercept: $\left(0,-\frac{1}{7}\right)$ Turning point: $\left(1,-\frac{1}{9}\right)$	
(iii)	$R_f=\left(-\infty,-\frac{1}{9}\right]\cup(1,\infty)$ $D_g=(-\infty,\infty)$ Hence gf exists since $R_f\subseteq D_g$.	
(iv)	**Scale** the graph **parallel to the *y*-axis** by a **factor of *a***. **Translate** the graph in the **positive *y*-direction** by ***b* units**.	
(v)	Since the horizontal line $y = a + b + 1$ will cut the curve of gf twice, gf is not one-one, hence, the inverse does not exist. Restriction is $D_f=[1,\infty)$	

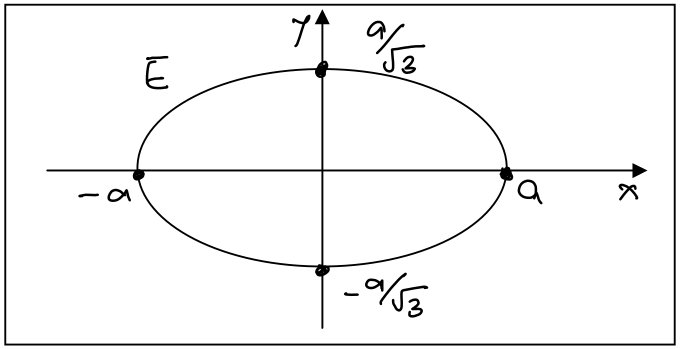
	<u>*Alternative Solution:*</u> Since the horizontal line $y = 2$ will cut the curve of f twice, f is not one-one. The curve of gf is a scaling and translation of f, hence the general shape of the graph is preserved and we can conclude that gf will also not be a one-one function. Hence the inverse does not exist.	

9(i)	The vector equation of l_2 is $\mathbf{r} = \quad \mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$ The normal vector of $\Pi_1 = \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} \times \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix}$ Scalar Product form of equation of Π_1 is $\mathbf{r} \bullet \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} = 6$ Cartesian equation of the plane Π_1 is $4x + 7y + 9z = 6$	
(ii)	Let d be the shortest distance. Then $d = \dfrac{\left\lvert \overrightarrow{AB} \bullet \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} \right\rvert}{\sqrt{9^2 + 7^2 + 4^2}}$, where $A = (1, -1, 1)$ and $B(1, -2, 0)$	

	$= \dfrac{\left\lvert \begin{pmatrix} 0 \\ -1 \\ -1 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} \right\rvert}{\sqrt{146}}$ $= \dfrac{16}{\sqrt{146}}$	
(iii)	Let θ be the angle between the two planes. $\cos\theta = \dfrac{\left\lvert \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} \bullet \begin{pmatrix} 4 \\ 7 \\ 9 \end{pmatrix} \right\rvert}{\sqrt{13}\sqrt{146}} = \dfrac{6}{\sqrt{13}\sqrt{146}}$ $\theta = 82.1°$	
(iv)	Note that (1, -1, 1) lies on Π_2 since $\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} = 1$ Also, note that l_1 is parallel to Π_2 since $\begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} \bullet \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} = 0$ So Π_2 contains l_1.	
(v)	Since both Π_1 and Π_2 contain l_1, l_1 is parallel to the plane $2x + y - az = b$	

	So $\begin{pmatrix} 2 \\ 1 \\ -a \end{pmatrix} \bullet \begin{pmatrix} 2 \\ -5 \\ 3 \end{pmatrix} = 0$ $\Rightarrow a = -\frac{1}{3}$ Note that (1, -1, 1) does not lie on plane $2x + y - az = b$ So 2(1) -1 –a (1) $\neq b \Rightarrow b \neq \frac{4}{3}$.	

10	$x = a\sin\theta \Rightarrow \frac{dx}{d\theta} = a\cos\theta$	
	$\int_{\frac{a}{2}}^{a} \sqrt{a^2 - x^2}\ dx = \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \sqrt{a^2 - (a\sin\theta)^2}\ (a\cos\theta)d\theta$ $= a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (\cos^2\theta)d\theta$	
	$= a^2 \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{(\cos 2\theta + 1)}{2} d\theta$	
	$= \frac{a^2}{2}\left[\frac{\sin 2\theta}{2} + \theta\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$	
	$= \frac{a^2}{2}\left[\frac{1}{2}\sin\frac{\pi}{2} + \frac{\pi}{2} - \frac{1}{2}\sin\frac{\pi}{3} - \frac{\pi}{6}\right] = \frac{a^2}{2}\left[\frac{\pi}{3} - \frac{1}{2}\left(\frac{\sqrt{3}}{2}\right)\right]$	

	$=-\frac{1}{8}\sqrt{3}a^2+\frac{\pi}{6}a^2=\frac{a^2}{24}\left(4\pi-3\sqrt{3}\right)$ (shown)	
10(i)	$x^2+3y^2=a^2 \Rightarrow \frac{x^2}{a^2}+\frac{y^2}{\left(\frac{a}{\sqrt{3}}\right)^2}=1$ 	
10(ii)	When $x=y,\ x^2+3x^2=a^2 \Rightarrow x=\frac{a}{2}$	
	Area of R = $\int_{\frac{a}{2}}^{a} y\ \mathrm{d}x+\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)=\frac{1}{\sqrt{3}}\int_{\frac{a}{2}}^{a}\sqrt{a^2-x^2}\ \mathrm{d}x+\frac{1}{2}\left(\frac{a}{2}\right)\left(\frac{a}{2}\right)$	
	$=\frac{1}{\sqrt{3}}\left[\frac{a^2}{24}\left(4\pi-3\sqrt{3}\right)\right]+\frac{a^2}{8}$	

	$=\frac{a^2}{24}\left[\left(\frac{4\pi-3\sqrt{3}}{\sqrt{3}}\right)+3\right]=\frac{a^2}{6}\left(\frac{\pi}{\sqrt{3}}\right)$ $k=\frac{\sqrt{3}}{18}$ or $k=\frac{1}{6\sqrt{3}}$	
10(iii)	Let $a=1 \Rightarrow x^2+3y^2=1$ Vol of S = $\pi\int_{\frac{1}{2}}^{\frac{1}{\sqrt{3}}} x^2 \ \mathrm{d}y+\frac{1}{3}\pi\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$ $=\pi\int_{\frac{1}{2}}^{\frac{1}{\sqrt{3}}}\left(1-3y^2\right) \ \mathrm{d}y+\frac{1}{3}\pi\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)$	
	= 0.162 units^3	