

**ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT**

**MATHEMATICS
Higher 2**

9740 / 02

Paper 2

15 September 2014

JC 2 PRELIMINARY EXAMINATION

Time allowed: **3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.

Write your Index number and full name on all the work you hand in.

Write in dark blue or black pen on your answer scripts.

You may use a soft pencil for any diagrams or graphs.

Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [] at the end of each question or part question.

At the end of the examination, fasten all your work securely together.

This document consists of **8** printed pages.



Anglo-Chinese Junior College

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2014

MATHEMATICS 9740
Higher 2
Paper 2

/ 100

Index No:

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Form Class: _____

Name: _____

Calculator model: _____

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/5
2	/7
3	/11
4	/7
5	/10
6	/2
7	/5
8	/6
9	/6
10	/8
11	/9
12	/9
13	/15

Summary of Areas for Improvement				
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Formula (F)	Presentation (P)

Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Formula (F)	Presentation (P)

Section A: Pure Mathematics [40 marks]

1 Do not use a calculator in answering this question.

Solve the inequality $\frac{2x+1}{x^2-x+9} \geq \frac{1}{x-3}$.

Hence find the range of values of u that satisfy the inequality $\frac{2u^2-1}{u^4+u^2+9} \leq \frac{1}{u^2+3}$. [5]

2 The complex number z satisfies the relations $\arg(z+3-3i) = -\frac{\pi}{4}$ and $|z-3+3i| \leq b$, where b is a constant and $1 < b < 3$.

(i) Illustrate each of the above relations on a single Argand diagram. [2]

(ii) Find the exact least possible value of $|z+5i|$. [1]

(iii) Given that the least possible value of $|z|$ is $\sqrt{18}-2$,

(a) find the value of b , [1]

(b) hence find an exact expression for z , in the form $x+iy$. [2]

(c) State the cartesian equation of the locus of the point representing complex variable w such that $|w| = |w-z_1|$, where z_1 is the complex number found in part (b). [1]

3 The function f is defined by

$$f: x \mapsto \frac{1}{p+(x-1)^2}, \text{ for } x \in \mathbb{R}, |x| \leq A, \text{ where } p \text{ is a constant and } p \geq 1.$$

(i) Find the largest value of A , for which f^{-1} exists. [1]

Use the value of A found in part (i) above for the rest of this question.

(ii) Find $f^{-1}(x)$ in terms of p , stating the domain of f^{-1} . [3]

The function g is defined by $g: x \mapsto e^{-\sqrt{x}}$, for $x \in \mathbb{R}, x \geq 0$.

(iii) By sketching the graphs of $y = g(x)$ and $y = g^{-1}(x)$, find the set of values of x for which $g(x) > g^{-1}(x)$. [3]

(iv) Show that the composite function $g^{-1}f$ exists. [2]

(v) Find the range of $g^{-1}f$ in terms of p . [2]

- 4** A loan company wishes to encourage its customers to repay their loans as soon as possible. By the middle of the month if the customer can repay part of the amount owed, then at the end of the month the company will charge an interest of 5%, only on the remaining amount still owed by the customer.

Mr Tan owes the company \$5000 at the beginning of the first month. He wishes to repay a fixed amount of \$ x , by the middle of every month until the loan is repaid in full.

- (i) Find the least value of x (to the nearest dollar) for which the total amount owed is paid off after the second payment. [2]
- (ii) Show that the value of x for which the whole amount owed is paid exactly after the $(n+1)^{\text{th}}$ payment is given by $x = \frac{5000r^n}{21r^n - 20}$, where r is to be determined. [3]
- (iii) If Mr Tan wishes to settle the amount owed in at most 10 payments what is the minimum amount (to the nearest dollar) he should pay each month. [2]

- 5** (a) By using the substitution $y = \frac{x}{t^3}$, find the general solution of the differential equation $t \frac{dy}{dt} - 3y(t-1) = (yt^2)^2$, and show that it can be expressed in the form $y = \frac{3}{t^3(Ae^{-3t} - 1)}$, where A is an arbitrary constant. [5]

State the equation of an asymptote that is common to all members of the family of solution curves.

Find an inequality that must be satisfied by A , for which a particular solution curve has only one vertical asymptote. [2]

- (b) The normal to any point on a given curve passes through the point (4, 8). Obtain a differential equation relating x and y . Verify that the general solution of this differential equation is of the form $x^2 + y^2 - 16y - 8x = A$, where A is an arbitrary constant such that $A > 0$.

Sketch a member of the family of solution curves. [3]

Section B: Statistics [60 marks]

- 6** A junior college has 50 male staff, 150 female staff, 800 male students and 1000 female students. The Vice-Principal wishes to take a feedback on the prices of the canteen food from a sample of 100 people.
- Explain how a quota sampling could be carried out to choose the 100 people. Explain briefly why this may not provide an accurate feedback to the Vice-Principal as compared to using a stratified sample. [2]
- 7** In an island resort, there are trams that go around the island which stopped over at many stations. The probability that a passenger on board will alight at Aquafront Station is 0.52. Assume that each passenger alights at a destination independently of one another.
- (i) On a particular weekday, there are 60 fully occupied trams, each with a seating capacity of 20 passengers, stopping over at Aquafront Station. Find the probability that the mean number of passengers alighting from a tram exceeds 11. [2]
- (ii) Due to the high demand of tram rides, new trams with seating capacity of 50 passengers are purchased to replace the smaller ones. Using a suitable approximation, find the probability that less than half of the passengers alighted from a fully occupied tram at Aquafront Station. [3]
- 8** The height X , in centimetres, of a randomly chosen girl in HTAM Junior College is normally distributed with mean μ cm and standard deviation σ cm.
- (i) Given that $P(X > \mu - a) + P(X > \mu + a) + P(\mu < X < \mu + 2a) = 1.38$, find the probability that the height of a randomly chosen girl is at least $2a$ cm more than the mean height. [2]
- (ii) It is known that 40% of the girls have heights which differ from μ cm by at most L cm.
- (a) Find L in terms of σ . [2]
- (b) Hence find the probability that the height of a randomly chosen girl exceeds μ cm by at least $2L$ cm. [2]

[Turn Over]

- 9 The table below shows the selling price of a particular smartphone T6 over a period of 6 months after its launch.

Month, x	1	2	3	4	5	6
Selling price, y dollars	180	169	149	121	86	52

- (i) Calculate the value of the product moment correlation coefficient, and explain why its value does not necessarily mean that the best model for the relationship between x and y is $y = a + bx$. [2]
- (ii) Draw a scatter diagram to illustrate the data, labelling the axes. [1]
- (iii) Explain how to use the values obtained by calculating product moment correlation coefficients to decide, for this data, whether $y = c + dx^2$ or $y = a + bx$ is the better model. [1]
- (iv) Estimate the time (in month) for the price of the smartphone to be \$20 using a suitable regression line. Comment on the reliability of your prediction. [2]
- 10 Express all your answers as proper fractions for this question.
- In 2013, 60% of the unemployed citizens of country A are graduates. A survey shows that within a year, 70% of these unemployed graduates found a job while only 40% of the unemployed citizens who are not graduates found a job.
- (i) A person who was unemployed in 2013 was selected at random. Find the probability that this person is not a graduate, given that he has not found a job. [3]
- (ii) Two persons who were unemployed in 2013 were selected at random. Find the probability that
- (a) exactly one of them has found a job within a year, [2]
- (b) exactly one of them is a graduate, given that both of them have found a job within a year. [3]

- 11** The mean mathematics examination mark for the whole school was 55 last year. Let the random variable X denote the mathematics examination mark obtained by a student this year.

- (i) A teacher observed that the mathematics examination marks obtained by 8 of her students this year are as follows: 25, 30, 35, 44, 50, 52, 58, 70.

Based on the results of these 8 students, carry out a test at the 5% significance level to determine whether the mean mark for the mathematics examination for the whole school this year is different from that of last year.

State appropriate hypotheses for the test, defining any symbols you use.

State a necessary assumption that you have made. [5]

- (ii) Another mathematics teacher has finished marking all the 120 scripts of the mathematics examination this year for his tutorial classes. The standard deviation of his 120 students' marks is 15. He carried out a test, at the 5% significance level, to determine whether the mean mark for the mathematics examination for the whole school this year has exceeded that of last year.

Find the range of values of \bar{x} for which the result of the test would be to reject the null hypothesis, where \bar{x} is the mean mark of his 120 students. (Answers obtained by trial and improvement from a calculator will obtain no marks.) [4]

- 12** The letters of the word ELEMENTARY are written on 10 cards with one letter on each card. Find the number of ways for which it is possible to

- (i) arrange the 10 cards in a row such that all vowels are together; [2]
(ii) arrange the 10 cards in a row such that all vowels are separated; [2]
(iii) select 5 cards with a total of 2 consonants and 3 vowels; [2]
(iv) arrange 4 cards in a row. [3]

[Turn Over]

- 13** For the convenience of the visitors to a mega theme park, two bus services, H1 and H2, are offered from the main entrance, to take the visitors to the different attractions in the theme park. Over a long period of time, it is found that the average number of H1 buses arriving at the entrance in a 10-minute period is 1 and the average number of H2 buses arriving at the entrance in a 12-minute period is 1. Assume that the buses arrive randomly and independently of one another.
- (i) State in this context, a condition that must be met for the numbers of buses of each service arriving at the main entrance to be well modelled by Poisson distributions. [1]
For the remaining of this question assume that the above condition is met.
- (ii) In a period of 10 minutes,
- (a) show that the probability that no buses arrive at the main entrance is 0.15988, correct to 5 significant figures. [2]
- (b) if there are exactly 3 buses arriving at the main entrance, find the probability that all 3 buses are H1 buses. [2]
- (iii) The transport manager wanted to check if there are sufficient buses arriving at the main entrance at every 10-minute non-overlapping intervals. Find the probability that more than 80% of the 10-minute non-overlapping intervals between 9.30am to 1.30pm have at least one bus service arriving at the main entrance. [3]
- (iv) In a period of t hours, the probability that there are at least 10 buses arriving at the main entrance is 0.8. Find the value of t , giving your answer correct to 2 decimal places. [3]
- (v) Use appropriate approximations, find the probability that there are more H2 buses arriving at the main entrance than there are H1 buses between 9.30am to 1.30pm. [4]

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