Anglo-Chinese Junior College

H2 Mathematics 9740

2014 JC 2 Preliminary Exam P2 Solution

Qn	Solution
1	

$$\frac{2x+1}{x^2-x+9} \geq \frac{1}{x-3}$$

$$\frac{2x+1}{x^2-x+9} - \frac{1}{x-3} \geq 0$$

$$\frac{(2x+1)(x-3)-(x^2-x+9)}{(x^2-x+9)(x-3)} \geq 0$$

$$\frac{(2x^2-5x-3)-(x^2-x+9)}{(x^2-x+9)(x-3)} \geq 0$$

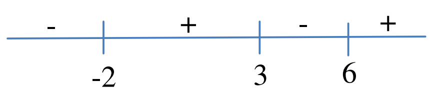
$$\frac{x^2-4x-12}{((x-\frac{1}{2})^2+\frac{35}{4})(x-3)} \geq 0$$

$$\frac{(x+2)(x-6)}{((x-\frac{1}{2})^2+\frac{35}{4})(x-3)} \geq 0$$

Since $(x-\frac{1}{2})^2+\frac{35}{4} > 0$ for all real values of x,

$$\frac{(x+2)(x-6)}{(x-3)} \geq 0$$

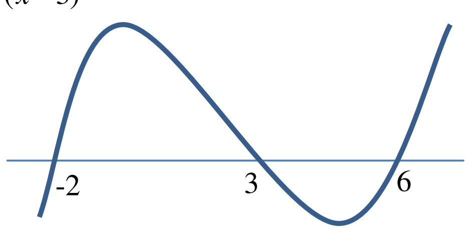
Method 1: Use of number line and sign test



$-2 \leq x < 3$ or $x \geq 6$

Method 2: Use of graphical method

$$\frac{(x+2)(x-6)}{(x-3)} \geq 0 \Leftrightarrow (x+2)(x-3)(x-6) \geq 0,\ x \neq 3$$



$-2 \leq x < 3$ or $x \geq 6$

Let $x = -u^2$

$$\frac{2u^2-1}{u^4+u^2+9} \leq \frac{1}{u^2+3}$$

$$\frac{-2x-1}{x^2-x+9} \leq \frac{1}{-x+3}$$

$$\frac{2x+1}{x^2-x+9} \geq \frac{1}{x-3}$$

Substituting $x = -u^2$ into solutions obtained above:

$-2 \leq x < 3$ or $x \geq 6$

$-2 \leq -u^2 < 3$ or $-u^2 \geq 6$

$-3 < u^2 \leq 2$ or $u^2 \leq -6$ (N.A.)

$u^2 \leq 2$ (since $u^2 > -3$ for all real values of u)

Method 1:

Using $a^2 \leq b^2 \Leftrightarrow |a| \leq |b|$,

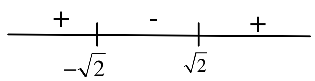
$u^2 \leq 2$

$-\sqrt{2} \leq u \leq \sqrt{2}$

Method 2:

$u^2 \leq 2$

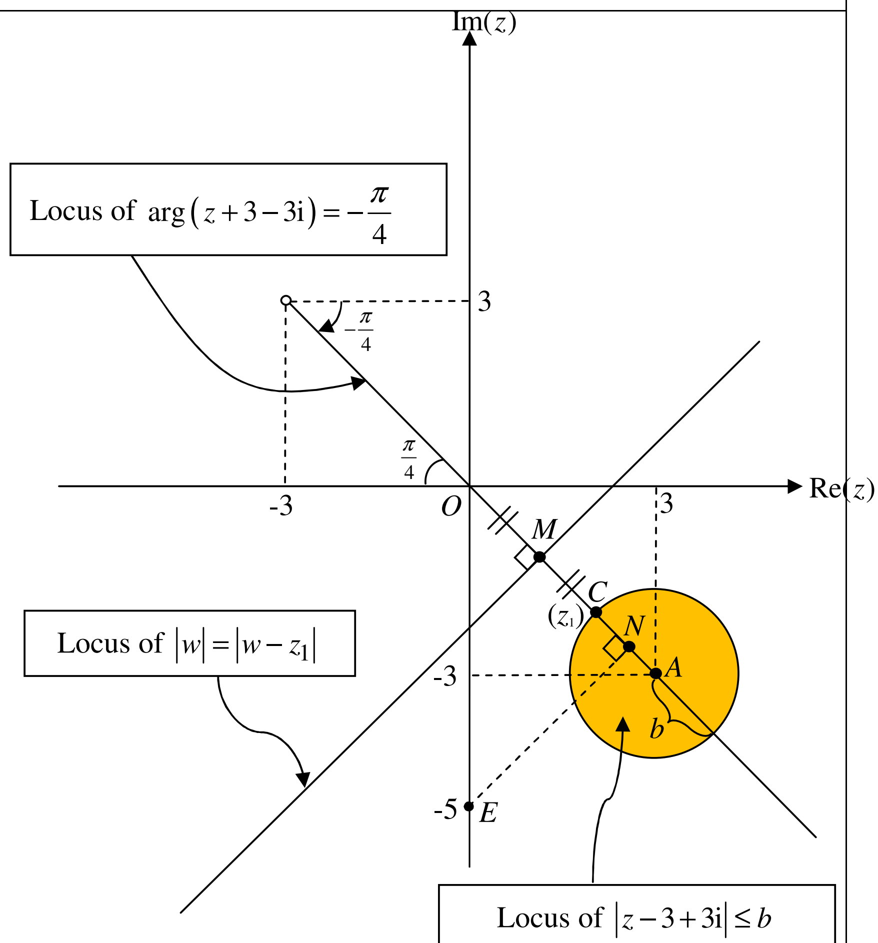
$u^2 - 2 \leq 0$



$(u-\sqrt{2})(u+\sqrt{2}) \leq 0$

$-\sqrt{2} \leq u \leq \sqrt{2}$

2 (i)



(ii)

Let d be the least possible value of $|z+5\mathrm{i}|$.

Using ΔONE,

$$\sin\frac{\pi}{4}=\frac{d}{5}$$

$$d=\frac{5}{\sqrt{2}}=\frac{5\sqrt{2}}{2}$$

(iii)
(a)

$$OA=\sqrt{3^2+3^2}=\sqrt{18}$$

$$b=OA-\left(\sqrt{18}-2\right)$$

$$b=2$$

(iii)
(b)

Method 1:

$$z=\left(\sqrt{18}-2\right)\left[\cos\left(-\frac{\pi}{4}\right)+i\sin\left(-\frac{\pi}{4}\right)\right]$$

$$=\left(3\sqrt{2}-2\right)\left[\frac{1}{\sqrt{2}}-\mathrm{i}\frac{1}{\sqrt{2}}\right]$$

$$=3-\frac{2}{\sqrt{2}}-\mathrm{i}\frac{\left(3\sqrt{2}-2\right)}{\sqrt{2}}$$

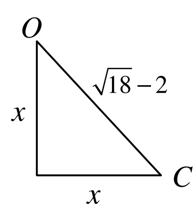
$$=3-\frac{2}{\sqrt{2}}-3\mathrm{i}+\mathrm{i}\frac{2}{\sqrt{2}}$$

$$=\left(3-\frac{2}{\sqrt{2}}\right)+\left(-3+\frac{2}{\sqrt{2}}\right)\mathrm{i}$$

$$=\left(3-\sqrt{2}\right)+\left(-3+\sqrt{2}\right)\mathrm{i}$$

Method 2:

$$x^2+x^2=\left(\sqrt{18}-2\right)^2$$



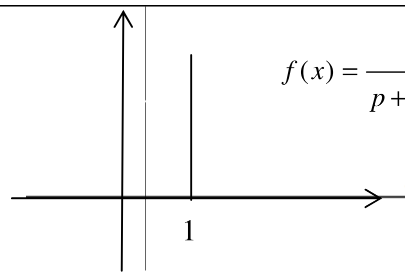
$$x^2=\frac{\left(\sqrt{18}-2\right)^2}{2}$$

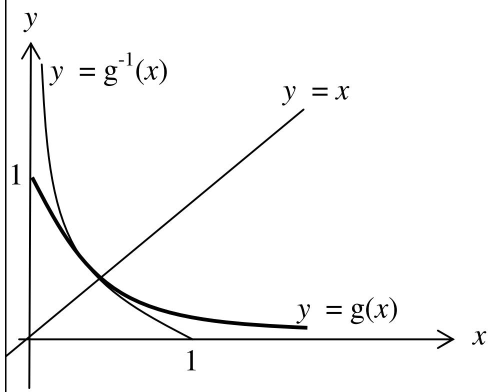
$$x=\sqrt{\frac{\left(\sqrt{18}-2\right)^2}{2}}$$

$$x=\frac{\sqrt{18}-2}{\sqrt{2}}$$

$$x=3-\sqrt{2}$$

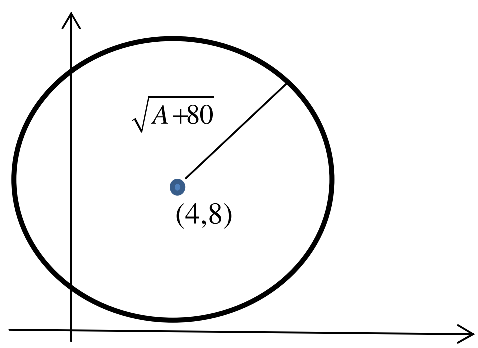
$$z=\left(3-\sqrt{2}\right)+\left(-3+\sqrt{2}\right)\mathrm{i}$$

(iii) (c)	The locus of $\lvert w\rvert=\lvert w-z_1\rvert$ is a perpendicular bisector of the line segment joining the points *O* and *C*. Since gradient of line segment *OC* is -1, Hence, gradient of perpendicular bisector = 1. Let point *M* be the midpoint of *OC*. Hence, point *M* is $\left(\frac{\left(3-\sqrt{2}\right)}{2}, \frac{\left(-3+\sqrt{2}\right)}{2}\right)$. Equation of locus: $y-\frac{\left(-3+\sqrt{2}\right)}{2}=(1)\left(x-\frac{\left(3-\sqrt{2}\right)}{2}\right)$ $y=x-\frac{\left(3-\sqrt{2}\right)}{2}+\frac{\left(-3+\sqrt{2}\right)}{2}$ $y=x-3+\sqrt{2}$
3 (i)	 Largest value of *A* =1
(ii)	Let $y=\frac{1}{p+(x-1)^2}$ $(x-1)^2=\frac{1}{y}-p$ $x=1\pm\sqrt{\frac{1}{y}-p}$ $x=1-\sqrt{\frac{1}{y}-p}\quad \sin ce\ -1\le x\le 1$ $\therefore\quad f^{-1}(x)=1-\sqrt{\frac{1}{x}-p}$ $D_{f^{-1}}=R_f=\left[\frac{1}{p+4}, \frac{1}{p}\right]$

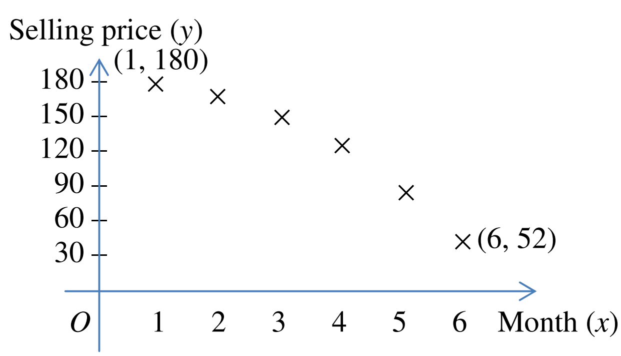
(iii)	$\mathrm{g}: x \mapsto e^{-\sqrt{x}}$, for $x \in \mathbb{R}, x \geq 0$.  Point of intersection between y=x and y=$g(x)$ is x= 0.495 Set of values of x for which $\mathrm{g}(x) > \mathrm{g}^{-1}(x)$ $\{x: 0.495 < x \leq 1\}$
(iv)	Check $R_f \subseteq D_{g^{-1}}$. *But* $D_{g^{-1}} = R_g = (0,1]$ $\therefore R_f = \left[\dfrac{1}{p+4}, \dfrac{1}{p}\right] \subseteq (0,1]$ since if $p \geq 1$, $\dfrac{1}{p+4} < \dfrac{1}{p} < 1$ $\therefore \mathrm{g}^{-1}\mathrm{f}$ exists.
(v)	Use $R_f = \left[\dfrac{1}{p+4}, \dfrac{1}{p}\right]$ as domain on $\mathrm{g}^{-1}(\mathrm{x})$ graph $\left[\dfrac{1}{p+4}, \dfrac{1}{p}\right]\left[\dfrac{1}{p+4}, \right]$ $g^{-1}\left(\dfrac{1}{p}\right) = a \Rightarrow g(a) = \dfrac{1}{p}$ *similarly* $g^{-1}\left(\dfrac{1}{p+4}\right) = \left(\ln(p+4)\right)^2$ $\therefore e^{-\sqrt{a}} = \dfrac{1}{p}$ $-\sqrt{a} = \ln\left(\dfrac{1}{p}\right)$ $a = (\ln p)^2$ Range of $\mathrm{g}^{-1}\mathrm{f} = \left[(\ln p)^2, \left(\ln(p+4)\right)^2\right]$
4 **(i)**	Amount owed at the start of 2^{nd} month is paid off after second payment : $(5000 - x)1.05 \leq x$ $x \geq \dfrac{5000(1.05)}{2.05}$ = \$2561 (nearest dollars)

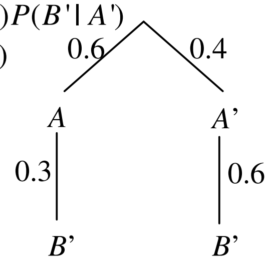
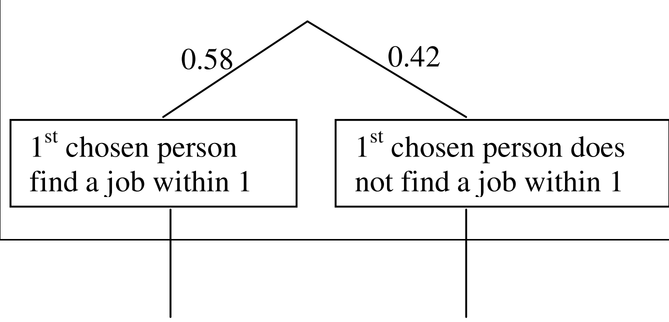
(ii)	Amount owed at the start of third month $=\left((5000-x)1.05-x\right)1.05 = 5000(1.05)^2 - (1.05^2+1.05)x$ Amount owed at the start of $(\text{n}+1)^{\text{th}}$ month $= 5000(1.05)^n - (1.05^n + 1.05^{n-1} + \ldots\ldots + 1.05)x$ $= 5000(1.05)^n - \dfrac{1.05x\left(1.05^n - 1\right)}{0.05}$ $= 5000(1.05)^n - 21x\left(1.05^n - 1\right)$ Amount owed is paid exactly after the $(\text{n}+1)^{\text{th}}$ payment: $5000(1.05)^n - 21x\left(1.05^n - 1\right) = x$ $x = \dfrac{5000(1.05)^n}{21\left(1.05^n - 1\right)+1} = \dfrac{5000(1.05)^n}{21\left(1.05^n\right)-20}$ $r = 1.05$
(iii)	At most 10 payments, $n = 9$ $x = \dfrac{5000(1.05)^9}{21\left(1.05^9\right)-20} = 616.69$ Miminum amount he should pay is \$617 per month
5 (a)	$y = \dfrac{\text{x}}{t^3} \Rightarrow yt^3 = x$ Differentiate w.r.t. t $3yt^2 + t^3\dfrac{dy}{dt} = \dfrac{dx}{dt}$ $t\dfrac{dy}{dt} = \dfrac{1}{t^2}\dfrac{dx}{dt} - 3y$ Substitute into given DE

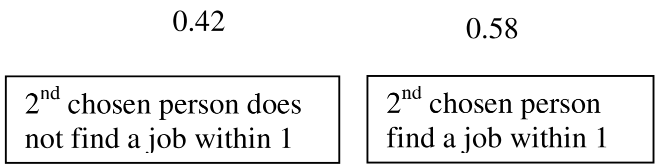
	$t\frac{dy}{dt}-3y(t-1)=\left(yt^2\right)^2$ $\frac{1}{t^2}\frac{dx}{dt}-3y-3y(t-1)=\left(yt^2\right)^2$ $\frac{dx}{dt}-3yt^2-3yt^2(t-1)=\left(yt^2\right)^2t^2$ $\frac{dx}{dt}-3yt^3=y^2t^6$ $\frac{dx}{dt}-3x=x^2$ $\frac{dx}{dt}=x^2+3x$ Solve this DE: $\int\frac{1}{x^2+3x}dx=\int 1dt$ $\int\frac{1}{x(x+3)}dx=t+C$ $\int\frac{1}{3x}-\frac{1}{3(x+3)}dx=t+C$ $\frac{1}{3}\ln\|x\|-\frac{1}{3}\ln\|x+3\|=t+C$ $\ln\left\|\frac{x}{x+3}\right\|=3t+C$ $\frac{x}{x+3}=\pm e^{3t+C}=Be^{3t}\quad where\ B=\pm e^C$ Make x the subject of the formula: $\frac{x}{x+3}=Be^{3t}$ $x=\frac{3Be^{3t}}{1-Be^{3t}}=\frac{3}{Ae^{-3t}-1}\quad where\ \frac{1}{B}=A$ $\therefore y=\frac{3}{t^3\left(Ae^{-3t}-1\right)}$ $As\ \ t\to 0,\ y\to\infty.$ Therefore y-axis or y=0 is an asymptote for all values of A. Denominator tends to 0 when $t=0$ or when $Ae^{-3t}-1=0$ $Ae^{-3t}-1=0$ $e^{-3t}=\frac{1}{A}$ has no solution if A is non-positive If a particular solution curve has only one vertical asymptote, then $A\le 0$
5 (b)	Gradient of normal at any point (x,y) on the curve is given by

	$m=-\frac{1}{dy/dx}$. Normal to any point (x,y) on a given curve passes through the point (4, 8), Equation of normal at point (x,y) is given by $y-8=m\,(x-4)$ $m=\frac{y-8}{x-4}$ $m=-\frac{1}{dy/dx}=\frac{y-8}{x-4}\Rightarrow\frac{dy}{dx}=\frac{4-x}{y-8}$ To verify that $x^2+y^2-16y-8x=A$ is a general solution Differentiate w.r.t. x and verify that DE is satisfied $2x+2y\frac{dy}{dx}-16\frac{dy}{dx}-8=0$ $\frac{dy}{dx}=\frac{8-2x}{2y-16}=\frac{4-x}{y-8}$ $x^2+y^2-16y-8x=A$ $(x-4)^2+(y-8)^2=A+80$ Sketch of any circle with centre at (4,8) and radius $\sqrt{A+80}$ for any positive value of arbitrary constant A. 
6	The Vice-principal will first decide on a quota to be fulfilled such as the one shown in the table below. <table><tr><td></td><td>Male</td><td>Female</td></tr><tr><td>Staff</td><td>25</td><td>25</td></tr><tr><td>Student</td><td>25</td><td>25</td></tr></table> Then, he will pick a sample of 100 people arbitrarily according to the quota given in the above table. Reason 1

	This method of sampling may not give an accurate feedback as compared to using a stratified sample because the sample may consist of a disproportionate number of staff (alternatively males) as compared to students (alternatively females). Reason 2 This method of sampling may not give an accurate feedback as compared to using a stratified sample because the staff and students chosen are not random and thus could be people chosen intentionally to provide a biased feedback.
7 (i)	Let X be the random variable "number of passengers alighting at Aquafront Station, out of 20." $X \sim \mathrm{B}(20,\ 0.52)$ $\mathrm{E}(X) = (20)(0.52) = 10.4$ $\mathrm{Var}(X) = (20)(0.52)(0.48) = 4.992$ $\mathrm{E}\left(\dfrac{X_1 + X_2 + X_3 + \ldots + X_{60}}{60}\right) = \dfrac{\mathrm{E}(X_1) + \mathrm{E}(X_2) + \ldots + \mathrm{E}(X_{60})}{60}$ $= \mathrm{E}(X) = 10.4$ $\mathrm{Var}\left(\dfrac{X_1 + X_2 + \ldots + X_{60}}{60}\right) = \dfrac{1}{60^2}\left[\mathrm{Var}\,(X_1) + \mathrm{Var}\,(X_2) + \ldots + \mathrm{Var}\,(X_{60})\right]$ $= \dfrac{60\mathrm{Var}\,(X)}{60^2} = \dfrac{4.992}{60} = 0.0832$ Since n is large, by Central Limit Theorem, $\overline{X} = \dfrac{X_1 + X_2 + X_3 + \ldots + X_{60}}{60} \sim N(10.4, 0.0832)$ approx $\mathrm{P}\,(\overline{X} > 11) = 0.0188$ (3 s.f)
7 (ii)	Let A be the random variable "number of passengers alighting at Aquafront Station, out of 50." $A \sim \mathrm{B}(50,\ 0.52)$ Since $n = 50$ is large, np = 50 (0.52) = 26 > 5 nq = 50 (0.48) = 24 > 5 $A \sim \mathrm{N}(26,\ 12.48)$ approx Probability that there are less than 25 passengers alighting at Aquafront Station $= \mathrm{P}\,(A < 25)$

	$\approx \mathrm{P}\,(A < 24.5)$ (continuity correction) $= 0.336$
8 (i)	$\mathrm{P}(X > \mu - a) + \mathrm{P}(X > \mu + a) + \mathrm{P}(\mu < X < \mu + 2a) = 1.38$ $1 + \mathrm{P}(\mu < X < \mu + 2a) = 1.38$ $\mathrm{P}(\mu < X < \mu + 2a) = 0.38$ $\mathrm{P}(X > \mu + 2a) = 0.5 - 0.38$ $\mathrm{P}(X > \mu + 2a) = 0.12$
8 (ii) (a)	$\mathrm{P}(\lvert X - \mu \rvert \le L) = 0.4$ $\mathrm{P}\left(\left\lvert \frac{X-\mu}{\sigma} \right\rvert \le \frac{L}{\sigma}\right) = 0.4$ $\mathrm{P}\left(-\frac{L}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{L}{\sigma}\right) = 0.4$ $\frac{L}{\sigma} = 0.5244005 = 0.524$ (to 3 s.f)
8 (ii) (b)	$\mathrm{P}(X - \mu \ge 2L) = \mathrm{P}\left(\frac{X-\mu}{\sigma} \ge \frac{2L}{\sigma}\right)$ $= \mathrm{P}\left(Z > \frac{2L}{\sigma}\right) = \mathrm{P}(Z > 2(0.5244005))$ $= \mathrm{P}(Z > 1.048801)$ $= 0.14713488 = 0.147$ (to 3 s.f)
9 (i)	$r = -0.98446 = -0.984$ (3s.f) Though the value of r show a strong negative linear correlation, it is possible that the sales price may have a curvilinear relationship.
9 (ii)	(a) 

9 (iii)	For $y = a + bx$, $r = -0.98446$. For $y = c + dx^2$, $r = -0.99874$ The model with $\lvert r \rvert$ closer to 1 gives a better model.
9 (iv)	Thus $y = 182.54237 - 3.7170795x^2$ [$y = 183 - 3.72x^2$ (to 3 s.f)] When y =20, $20 = 182.54237 - 3.7170795x^2$ $x = 6.61275 = 6.61$ (to 3s.f) Even though $r = -0.99874$ is close to –1, which shows a strong negative linear correlation, but it is not reliable since y = 20 is (an extrapolation) not within the data range.
10 **(i)**	Let A = {a randomly chosen unemployed person in 2013 is a graduate} Let B = {a randomly chosen unemployed person in 2013 found a job within a year} Given $P(A) = 0.6$, $P(B \mid A) = 0.7$, $P(B \mid A') = 0.4$. $P(B') = P(A)P(B' \mid A) + P(A')P(B' \mid A')$ $= (0.6)(0.3) + (0.4)(0.6)$ $= 0.18 + 0.24$ $= 0.42$ $= \dfrac{21}{50}$  $P(A' \mid B') = \dfrac{P(A' \cap B')}{P(B')}$ $= \dfrac{P(A')P(B' \mid A')}{P(B')}$ $= \dfrac{0.24}{0.42} = \dfrac{4}{7}$
(ii) **(a)**	From (ii)(a), we can find that P(B) = 0.58 



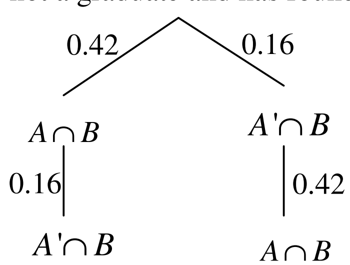
P(exactly one of them has found a job within a year)
$= (0.58)(0.42) + (0.42)(0.58) = 2(0.58)(0.42)$
$= 0.4872 = \dfrac{609}{1250}$

(b)

Let C = {Out of 2 randomly chosen persons who were unemployed in 2013, exactly one is a graduate}
Let D = {Out of 2 randomly chosen persons who were unemployed in 2013, both of them have found a job within 1 year}

Hence, the event
$C \cap D$ = {Out of 2 random chosen people who were unemployed in 2013, one person is a fresh graduate and he has found a job within a year and the other is not a graduate and has found a job within a year}

Note:
{a randomly chosen person who is unemployed in the year 2013 is a fresh graduate and he has found a job within a year}
$A' \cap B$ = {a randomly chosen person who is unemployed in the year 2013 is not a graduate and has found a job within a year}



$P(C \cap D)$
$= (0.42)(0.16) + (0.16)(0.42)$
$= 2(0.42)(0.16)$
$= 0.1344$
$= \dfrac{84}{625}$

$P(D) = (0.58)(0.58) = 0.3364$

$$P(C \mid D) = \frac{P(C \cap D)}{P(D)} = \frac{0.1344}{0.3364} = \frac{336}{841}$$

11 (i)

Let random variable X be a student's mathematics examination mark for this year.

Let $\overline{X} = \dfrac{X_1 + X_2 + X_3 + \ldots + X_n}{n}$

To test H_0: $\mu = 55$
against H_1: $\mu \neq 55$ at the 5% significance level,
where μ is the population **mean** mark of students in the mathematics examination **this year**.

Assume that X is normally distributed.
Since $n = 8 \leq 30$ and population variance is unknown.

Under H_0, $T = \dfrac{\overline{X} - 55}{15.0807/\sqrt{8}} \sim t(7)$

From G.C.,
$t = -1.781747209$,
p-value: $p = 0.1179931625 > 0.05$

There is insufficient evidence to conclude that the mean mark of the mathematics examination for the whole school this year is different from that of last year's.

(ii)

To test H_0: $\mu = 55$
against H_1: $\mu > 55$ at the 5% significance level,
where μ is the population mean mark of students in the mathematics examination this year.
Since $n = 120$ is large, by Central Limit Theorem,

Under H_0, $Z = \dfrac{\overline{X} - 55}{S/\sqrt{n}} \sim N(0,1)$ approximately .

Given sample variance = 15^2.
Unbiased estimate for σ^2:

$$s^2 = \frac{n}{n-1} \times \text{sample variance}$$

$$s^2 = \frac{120}{119} \times 15^2 = 226.8907563$$

To reject H_0, z must be in the critical region. Thus,

$$z > 1.645$$

$$\frac{\bar{x} - 55}{s/\sqrt{n}} > 1.645$$

$$\bar{x} > 55 + 1.645 \frac{s}{\sqrt{n}}$$

$$\bar{x} > 55 + 1.645 \frac{\sqrt{\frac{120}{119}(15^2)}}{\sqrt{120}}$$

$$\bar{x} > 55 + 1.645 \frac{15}{\sqrt{119}}$$

$$\bar{x} > 57.3$$

12

There are only 3 identical letters E.

There are 7 distinct letters L, M, N, T, A, R, Y.

There are 4 vowels of which 3 are identical letters: A, E, E, E

There are 6 distinct consonants: L, M, N, T, R, Y

(i)

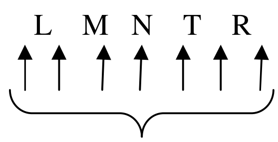
Step 1: Arrange the 6 distinct consonants and the group of 4 vowels (treated as 7 objects) in a row: 7!
Step 2: Arrange the vowels among themselves: 4!/3!

Ans: $7! \times \frac{4!}{3!} = 20160$

(ii)

Step 1: Arrange the 6 distinct consonants in a row: 6!

Step 2: Choose 4 positions out of the 7 positions indicated by the arrows in the diagram below to insert the 4 vowels: ${}^{7}C_{4}$



Insert the 4 vowels A, E, E, E into 4 of these 5 positions between the vowels and the 2 ends (total of 7 positions).

Step 3: Arrange the 4 vowels (A, E, E, E) in the 4 chosen positions in step 2: $\frac{4!}{3!}$

Ans: $6! \times {}^{7}C_{4} \times \frac{4!}{3!} = 100800$

(iii)

Case 1: 3 vowels are E, E, E
Step 1: Choose 2 consonants out of the 6 consonants: ${}^{6}C_{2}$
Step 2: Choose all the 3 E's: 1

Case 2: 3 vowels are A, E, E
Step 1: Choose 2 consonants out of the 6 consonants: ${}^{6}C_{2}$
Step 2: Choose any 2 E's: 1
Step 3: Choose the only A: 1

Ans: ${}^{6}C_{2} \times 1 + {}^{6}C_{2} \times 1 \times 1 = 30$

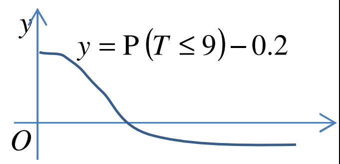
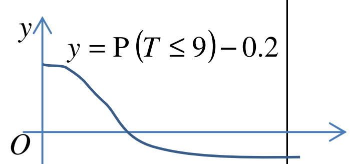
(d)

Case 1: 3 E's and 1 other letter
Step 1: Choose 3 E's: 1
Step 2: Choose one other letter that is not an E: 7
Step 3: Arrange the 4 chosen letters with 3 identical E's: $\frac{4!}{3!}$

Case 2: 2 E's and 2 other distinct letters
Step 1: Choose 2 E's: 1
Step 2: Choose 2 other distinct letters that are not E: ${}^{7}C_{2}$
Step 3: Arrange the 4 chosen letters with 3 identical E's: $\frac{4!}{2!}$

Case 3: 4 distinct letters (may be broken into a case with 1 E and another case with no E)

	Step 1: Choose 4 distinct letters from the 8 letters (A, E, L, M, N, T, A, R, Y): ${}^8C_4$ Step 2: Arrange the 4 chosen distinct letters: 4! Ans: $1\times 7\times\frac{4!}{3!}+1\times {}^7C_2\times\frac{4!}{2!}+{}^8C_4\times 4!=1960$
13	
(i)	The <u>average</u> number of H1 buses arriving in any 10-minute period is **constant** at 1 and the <u>average</u> number of H2 buses arriving in any 12-minute period is **constant** at 1.
(ii) **(a)**	Let X be the random variable "number of buses from bus service H1 in 10 minutes" $X \sim \text{Po}(1)$ Let Y be the r.v "number of buses from bus service H2 in 10 minutes" $Y \sim \text{Po}\left(\frac{1}{12}\times 10\right)$ Required probability $= \text{P}(X=0)\times\text{P}(Y=0)$ = (0.36787944)(0.4345982)= 0.15987975 = 0.15988 (shown)
(ii) **(a)**	**OR** $X+Y \sim \text{Po}\left(1+\frac{1}{12}\times 10\right)$ $X+Y \sim \text{Po}\left(\frac{11}{6}\right)$ Required probability $= \text{P}(X+Y=0)$ = 0.15987975 = 0.15988 (shown)
(ii) **(b)**	Required probability $= \text{P}(X=3 \mid X+Y=3)$ $= \frac{\text{P}(X=3)\times\text{P}(Y=0)}{\text{P}(X+Y=3)}$ $= \frac{0.061313\times 0.434598}{\text{P}\,0.164197}$ = 0.162284 = 0.162 (to 3 s.f)
(iii)	Let A be the random variable "number of 10-minute intervals with at least one bus service at the main entrance boarding point, out of 24 10-minute intervals". $A \sim \text{B}(24,\ 1-0.15987975)$ $A \sim \text{B}(24,\ 0.8401202539)$

	$\mathrm{P}(A > 19.2) = \mathrm{P}(A \geq 20)$ $= 1 - \mathrm{P}(A \leq 19)$ $= 0.66403$ $= 0.664$ (to 3 s.f)
(iv)	Let T be the random variable "total number of buses from both bus services in t hours" $T \sim \mathrm{Po}\left(\frac{2.2 \times 60}{12} t\right) \Rightarrow T \sim \mathrm{Po}(11\ t)$ $\mathrm{P}(T \geq 10) = 0.8$ $1 - \mathrm{P}(T \leq 9) = 0.8$ $\mathrm{P}(T \leq 9) - 0.2 = 0$ Solve using GC,  $q = 68.284106$ minutes $t = 1.13807$ hrs $= 1.14$ hrs (to 2 d.p)
	OR Let T be the r.v "total number of buses from both bus services in q min" $T \sim \mathrm{Po}\left(\frac{2.2}{12} q\right) \Rightarrow T \sim \mathrm{Po}\left(\frac{11}{60} q\right)$ $\mathrm{P}(T \geq 10) = 0.8$ $1 - \mathrm{P}(T \leq 9) = 0.8$ $\mathrm{P}(T \leq 9) - 0.2 = 0$ Solve using GC,  $q = 68.284106$ minutes $\therefore\ t = 1.1381$ hrs $= 1.14$ hrs (to 2 d.p)
(v)	Let C be the random variable "number of buses from bus service H1 in 240 minutes" $C \sim \mathrm{Po}\left(\frac{1}{10} \times 240\right) \Rightarrow C \sim \mathrm{Po}(24)$ Since $\lambda = 24 > 10$, $C \sim \mathrm{N}(24, 24)$ approx. Let D be the r.v "number of buses from bus service H2 in 240 minutes"

	$D \sim \text{Po}(20)$ Since $\lambda = 20 > 10$, $D \sim \text{N}(20,\ 20)$ approx. $D - C \sim \text{N}(-4,\ 44)$ approx Probability that there are more buses of bus service H2 than of bus service H1 between 9.30am to 1.30pm = $\text{P}(D > C)$ $= \text{P}(D - C > 0)$ $= \text{P}(D - C > 0.5)$ (continuity correction) = 0.2487588 = 0.249 (3 s.f)