ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT

MATHEMATICS
Higher 2

9740 / 01

Paper 1

26 August 2014

JC 2 PRELIMINARY EXAMINATION

Time allowed**: 3 hours**

Additional Materials: List of Formulae (MF15)

READ THESE INSTRUCTIONS FIRST

Write your Index number, Form Class, graphic and/or scientific calculator model/s on the cover page.
Write your Index number and full name on all the work you hand in.
Write in dark blue or black pen on your answer scripts.
You may use a soft pencil for any diagrams or graphs.
Do not use paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in the question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

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ANGLO-CHINESE JUNIOR COLLEGE
MATHEMATICS DEPARTMENT
JC2 Preliminary Examination 2014

MATHEMATICS 9740
Higher 2
Paper 1

/ 100

Index No: | | | | | Form Class: ___________
Name: ___________________________

Calculator model: ___________________

Arrange your answers in the same numerical order.

Place this cover sheet on top of them and tie them together with the string provided.

Question No.	Marks
1	/5
2	/7
3	/9
4	/10
5	/8
6	/13
7	/15
8	/11
9	/11
10	/11

Summary of Areas for Improvement				
Knowledge (K)	Careless Mistakes (C)	Read/Interpret Qn wrongly (R)	Formula (F)	Presentation (P)

1 Referred to the origin O, the points A and B lie in the same plane such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point M on OA is such that $OM : MA = 1 : 2$, and the point N on OB is such that $ON : OB = 1 : 3$. The point S is on AB such that $5AS = 4SB$.

Write down the position vector of S in terms of $\mathbf{a}$ and $\mathbf{b}$.

Using vector product, find the ratio of the area of triangle MNS to the area of triangle OAB. [5]

2 P is a variable point on the circumference of a circle with diameter AB, and Q is the point on AB such that $AQ = AP$. Given that angle $PAQ = \theta$ radians and $AB = \ell$, show that the area S of triangle PAQ is given by $S = \frac{1}{2}\ell^2\left(\sin\theta - \sin^3\theta\right)$. [3]

Use differentiation to find, in surd form and in terms of ℓ, the maximum value of S, proving that it is a maximum. [4]

3 The curve $y = \mathrm{f}(x)$ passes through the point $(0,1)$ and satisfies the equation $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6-2y}{\cos 2x}$.

Find the Maclaurin's series of $\mathrm{f}(x)$, up to and including the term in x^3. [4]

Using standard results given in the List of Formulae (MF15), express $\frac{1-\sin x}{\cos x}$ as a power series of x, up to and including the term in x^3. [3]

Using the two power series you have found, show to this degree of approximation, that $\mathrm{f}(x)$ can be expressed as $a(\tan 2x - \sec 2x) + b$ where a and b are constants to be determined. [2]

4 **(i)** One of the roots of the equation $z^3 = (-2+2\mathrm{i})$ is $1+\mathrm{i}$. Find the remaining roots in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$. [3]

(ii) Given that $1+\mathrm{i}$ is a root of the equation $2w^3 + aw^2 + bw - 2 = 0$, find the values of the real numbers a and b. [4]

(iii) For these values of a and b, solve the equation in part **(ii)**, without the use of a calculator. [3]

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5 Planes p_1, p_2 and p_3 have equations

$$-2x + z = 4,$$
$$2x + y - 2z = 6,$$
$$-6x + 4y + \lambda z = \mu$$

respectively, where λ and μ are constants.

(i) Find a vector parallel to both p_1 and p_2. [2]

Given that the point with coordinates $(-5, \alpha, \beta)$ lies on p_1 and p_2, find α and β.

Hence find a vector equation of the line of intersection of p_1 and p_2. [3]

(ii) Given that p_1, p_2 and p_3 form a triangular prism, what can be said about the values of λ and μ? [3]

6 It is given that $\mathrm{f}(x) = \dfrac{x^2 - 4c^2}{x - c}$, where c is a positive constant.

Sketch the graph of $y = \mathrm{f}(x)$, giving the equations of any asymptotes and the coordinates of any points of intersection with the axes. [3]

Hence

(i) state the set of values of k for which the equation $x^2 - 4c^2 = k(x - c)$ has two distinct positive roots, [1]

(ii) describe fully a sequence of transformations which would transform the graph of $y = \mathrm{f}(x)$ to the graph of $y = \dfrac{4x(x + 4c)}{x + c}$, [2]

(iii) on separate diagrams, sketch the graphs of

(a) $y = \mathrm{f}'(x)$, **(b)** $y = \dfrac{x - c}{x^2 - 4c^2}$, **(c)** $y = \dfrac{4c^2 - x^2}{|x| + c}$,

giving the equations of any asymptotes and the coordinates of any points of intersection with the axes. [7]

7 **(a)** Prove by the method of mathematical induction that

$$\sum_{r=1}^{n} \frac{1}{r(r+1)(r+2)} = \frac{1}{4} - \frac{1}{2(n+1)(n+2)}. \qquad [4]$$

Express $\dfrac{5n+3}{n(n+1)(n+2)(n+3)}$ in the form $\dfrac{a}{n(n+1)(n+2)} + \dfrac{b}{(n+1)(n+2)(n+3)}$,

where a and b are constants to be determined. [1]

Hence evaluate $\displaystyle\sum_{r=1}^{n} \frac{5r+3}{r(r+1)(r+2)(r+3)}$. [4]

(b) Show that $\dfrac{1}{n-2} - \dfrac{1}{n+2} = \dfrac{C}{n^2-4}$, where C is a constant to be found. [1]

Hence find $\displaystyle\sum_{r=3}^{n} \frac{1}{r^2-4}$. (There is no need to express your answer as a single algebraic fraction.) [3]

Give a reason why the series $\displaystyle\sum_{r=3}^{\infty} \frac{1}{r^2-4}$ converges, and write down its value. [2]

8 The planes p_1 and p_2 have equations $\mathbf{r}.\begin{pmatrix} -1 \\ -2 \\ 2 \end{pmatrix} = -1$ and $\mathbf{r}.\begin{pmatrix} -7 \\ 4 \\ 4 \end{pmatrix} = 1$ respectively.

(i) Find the acute angle between p_1 and p_2. [2]

(ii) The point A (2 , α , 3) is equidistant from the planes p_1 and p_2. Calculate the two possible values of α. [5]

(iii) Find the position vector of the foot of perpendicular from B (0 ,1 , 2) to the plane p_1. Hence find the cartesian equation of the plane p_3 such that p_3 is parallel to p_1 and point B is equidistant from planes p_1 and p_3. [4]

[Turn Over

9 A curve C has parametric equations $x = 2t, \quad y = \dfrac{2}{t}$.

(i) Find the equation of the tangent to C at the point with parameter t. [3]

(ii) What can be said about the normals to C as $t \to \pm\infty$? [1]

(iii) Points P and Q on C have parameters p and q respectively. The tangent at P meets the tangent at Q at the point R. Show that the x-coordinate of R is $\dfrac{4pq}{p+q}$, and find the y-coordinate of R in terms of p and q. [3]

It is given that $pq = 1$.

(iv) State the cartesian equation of the locus of R, as p and q vary. [1]

(v) Given also that $p = 2$, without finding the cartesian equation of C, find the exact value of the area of the region bounded by C and the tangents at P and Q. [3]

10 **(i)** Using the substitution $y = \ln x$, show that $\int (\ln x)^2 \, \mathrm{d}x = \int y^2 \mathrm{e}^y \mathrm{d}y$. [1]

Hence show that $\int (\ln x)^2 \, \mathrm{d}x = x\left\{(\ln x)^2 - 2(\ln x) + 2\right\} + c$. [3]

(ii) It is given that

$$\mathrm{f}(x) = \begin{cases} \ln x & \text{for } 1 \le x \le \mathrm{e}, \\ 0 & \text{for } \mathrm{e} < x \le 3, \end{cases}$$

and that $\mathrm{f}(x+2) = \mathrm{f}(x)$ for all real values of x.

(a) The region bounded by the curve $y = \mathrm{f}(x)$, the x-axis and the line $x = \mathrm{e}$ is rotated through 2π radians about the x-axis. Find the exact value of the volume obtained. [2]

(b) Sketch the graph of $y = \mathrm{f}(x)$ for $-3 \le x \le 4$. [2]

Hence find $\int_{-3}^{4} \mathrm{f}(x) \mathrm{d}x$. [3]

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