Chapter 1

Solving Simultaneous Equations

1.1 Solving Simultaneous Equations involving one Linear and one Non - Linear Equation in Two Unknown Variables

Solving two simultaneous equations in two unknowns x and y, means finding the values of x and y that satisfy both equations.

When solving simultaneous equations in two unknown variables involving one linear and one non-linear equation, we use the **method of substitution**.

Step 1: Rearrange the linear equation to get $x = \ldots$ or $y = \ldots$.
Step 2: Substitute the resulting equation in Step 1 into the non-linear equation to form another equation with one variable, then solve for the variable.
Step 3: Substitute back the value obtained in Step 2 into the linear equation and solve for the other variable.
Step 4: Check your solutions in both of the original equations.

Example 1

Solve the simultaneous equations

$$x + y = 2,$$
$$y = x^2 + 2x - 8.$$

Solution

Let
$$x + y = 2 \ldots\ldots(1)$$
$$y = x^2 + 2x - 8 \ldots\ldots(2)$$

From (1)
$$y = 2 - x \ldots\ldots(3)$$

Substituting (3) into (2) gives
$$2 - x = x^2 + 2x - 8$$
$$x^2 + 3x - 10 = 0$$
$$(x + 5)(x - 2) = 0$$
$$\therefore \quad x = 2 \quad \text{or} \quad x = -5$$

Substituting $x = 2$ into (3) gives $y = 0$.
Substituting $x = -5$ into (3) gives $y = 7$.

$\therefore x = 2,\ y = 0$ or $x = -5,\ y = 7$.

Try 1

Solve the simultaneous equations

$$y - 4x^2 = 0,$$
$$y + 2x = 2.$$

Example 2

Solve the simultaneous equations

$$27x^2+5y^2=12(xy+1),$$
$$9x-5y=8$$

Solution

$$27x^2+5y^2=12(xy+1)\ldots\ldots\ldots\ldots\ldots(1)$$
$$9x-5y=8\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

From (2)

$$9x=8+5y$$
$$x=\frac{8+5y}{9}\ldots\ldots\ldots\ldots\ldots\ldots(3)$$

Substituting (3) into (1):

$$27\left(\frac{8+5y}{9}\right)^2+5y^2=12\left[\left(\frac{8+5y}{9}\right)y+1\right]$$
$$27\left(\frac{8+5y}{9}\right)^2+5y^2=\frac{4}{3}y(8+5y)+12$$
$$\frac{1}{3}(64+80y+25y^2)+5y^2=\frac{4}{3}y(8+5y)+12$$
$$64+80y+25y^2+15y^2=4y(8+5y)+36$$
$$40y^2+80y+64=32y+20y^2+36$$

Substituting $y=-1$ into (3):

$$x=\frac{8+5(-1)}{9}$$
$$=\frac{1}{3}$$

$$20y^2+48y+28=0$$
$$4(5y^2+12y+7)=0$$
$$4(5y+7)(y+1)=0$$
$$y=-\frac{7}{5}\quad\text{or}\quad y=-1$$

Substituting $y=-\frac{7}{5}$ (1):

$$x=\frac{8+5\left(-\frac{7}{5}\right)}{9}$$
$$=\frac{1}{9}$$

$$\therefore\ x=\frac{1}{3},\ y=-1;\ x=\frac{1}{9},\ y=-\frac{7}{5}$$

Try 2

Solve the simultaneous equations

$$\frac{4x}{y}+\frac{9y}{x}=15,$$
$$2y+3x=1.$$

1.2 Finding the Points of Intersection between a Line and a Curve

We can find the points of intersection of two graphs by solving both equations simultaneously.

Example 3

Find the coordinates of the points of intersection of the graphs whose equations are $x - y + 6 = 0$ and $x^2 + y^2 - 6x - 8y = 0$.

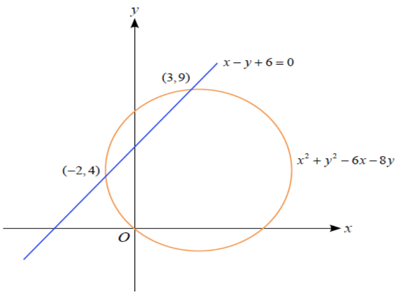
Find the coordinates of the points of intersection of the graphs whose equations are $x - y + 6 = 0$ and $x^2 + y^2 - 6x - 8y = 0$.

Let $x - y + 6 = 0$..................................(1)

$x^2 + y^2 - 6x - 8y = 0$......................(2)

From (1)

$y = x + 6$..(3)



Substituting (3) into (2) gives

$$x^2 + (x+6)^2 - 6x - 8(x+6) = 0$$
$$x^2 + x^2 + 12x + 36 - 6x - 8x - 48 = 0$$
$$2x^2 - 2x - 12 = 0$$
$$2(x^2 - x - 6) = 0$$
$$2(x+2)(x-3) = 0$$

$$x = -2 \quad \text{or} \quad x = 3$$

Substituting $x = -2$ and $x = 3$ into (3) gives $y = 4$ and $y = 9$ respectively.

` $\therefore$ the coordinates of the points of intersections are $(-2,\ 4)$ and $(3,\ 9)$.

Try 3

Find the coordinates of the points of intersection of the graphs whose equations are

$\frac{3}{2x} - \frac{1}{y} = 1$ and $6x + y = 8$.

1.4 Solving word Problems using Simultaneous Equations

The principle of solving simultaneous equations can also be applied to solve word problems.
The steps to solve such problems can be summarised as follows.

Step 1:	Analyse the facts in the problem.
Step 2:	Use variables to represent the unknown quantities.
Step 3:	Formulate two equations using these variables.
Step 4:	Solve the equations simultaneously.

Example 4

The perimeter of a rectangle is 22 cm and its area is 28 cm^2. Find its length and breadth, where the length is longer the breadth.

Solution

Let the length and breadth of the rectangle be x cm and y cm respectively.

Given Perimeter $= 22$ cm

$\therefore \quad 2x + 2y = 22$

$x + y = 11$

$y = 11 - x$...(1)

Given Area $= 28$ cm^2

$\therefore \quad xy = 28$..(2)

Substitute (1) into (2):

$x(11 - x) = 28$

$x^2 - 11x + 28 = 0$

$(x-4)(x-7) = 0$

$x = 4$ or $x = 7$

Substitute $x = 4$ into (1):

$y = 11 - 4$

$= 7$

Substitute $x = 7$ into (1):

$y = 11 - 7$

$= 4$

$\therefore$ the length of the rectangle is 7 cm and the breadth is 4 cm.

Try 4

A rectangular mirror has an area of 350 cm^2. Given that twice its length is 8 cm longer than three times its width. Find the dimensions of this mirror.